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A THEORY OF THE INTERSTELLAR MEDIUM: THREE COMPONENTS REGULATED BY SUPERNOVA EXPLOSIONS IN AN INHOMOGENEOUS SUBSTRATE

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ABSTRACT

Supernova explosions in a cloudy interstellar medium produce a three-component medium in which a large fraction of the volume is filled with hot, tenuous gas. In the disk of the galaxy the evolution of supernova remnants is altered by evaporation of cool clouds embedded in the hot medium. Radiative losses are enhanced by the resulting increase in density and by radiation from the conductive interfaces between clouds and hot gas. Mass balance (cloud evaporation rate = dense shell formation rate) and energy balance (supernova shock input = radiation loss) determine the density and temperature of the hot medium with $(n, T) = (10^{-2.5}, 10^{5.7})$ being representative values. Very small clouds will be rapidly evaporated or swept up. The outer edges of "standard" clouds ionized by the diffuse UV and soft X-ray backgrounds provide the warm ($\sim 10^4$ K) ionized and neutral components. A self-consistent model of the interstellar medium developed herein accounts for the observed pressure of interstellar clouds, the galactic soft X-ray background, the O VI absorption line observations, the ionization and heating of much of the interstellar medium, and the motions of the clouds. In the halo of the galaxy, where the clouds are relatively unimportant, we estimate $(n, T) = (10^{-3.3}, 10^{6.0})$ below one pressure scale height. Energy input from halo supernovae is probably adequate to drive a galactic wind.

Subject headings: interstellar: matter — nebulae: supernova remnants — X-rays: general

I. INTRODUCTION

Recent observations have drastically modified our picture of the interstellar medium; measurements of the soft X-ray background (e.g., Burstein *et al.* 1977) and of ubiquitous O VI absorption lines (Jenkins and Meloy 1974; York 1974) both suggest the presence of a large amount of hot, low-density gas in the disk of our Galaxy.

From a theoretical point of view as well it has become increasingly clear that supernova explosions must generate large volumes of hot, low-density gas which are very difficult to destroy once they are created. The point, originally made by Cox and Smith (1974), who studied supernova explosions in a uniform medium, is strengthened and altered when one considers supernovae exploding in an extremely inhomogeneous cloudy medium, because the supernova shocks tend to propagate between the clouds in the less resistive component. Although the dynamics are dominated by interactions between the remnant and interstellar clouds, we will first review the case of expansion in a uniform medium.

From the work of Chevalier (1974) we estimate that a supernova remnant (SNR) with a total energy $E = 10^{51}E_{51}$ ergs in a uniform medium with a hydrogen density n_0 would expand adiabatically to a radius $R_c = 10^{1.29}E_{51}^{0.29}n_0^{-0.41}$ pc, where approximately one-half the mass accumulates in a cool dense shell and the internal pressure drops by a factor of about 2 to $P_c' = \frac{1}{2}P_c = E/(4\pi R_c^3)$.[1] Subsequently the SNR expands with roughly constant momentum (Chevalier finds $R \propto t^{0.31}$) until the internal pressure P drops to the ambient interstellar pressure P_0. This maximum radius $R_E = R_c(P_c'/P_0)^{1/5} = 10^{1.74}E_{51}^{0.32}n_0^{-0.16}\tilde{P}_{04}^{-0.20}$ is reached[2] at a time $t_E = 10^{5.92}E_{51}^{0.31}n_0^{0.27}\tilde{P}_{04}^{-0.64}$ yr, where $\tilde{P}_{04} = 10^{-4}P_0/k$. The hot, low-density cavity created by the SNR will survive for approximately the time $t_{\max} = R_E/(P_0/\rho_0)^{1/2} = 10^{6.85}E_{51}^{0.32}n_0^{0.34}\tilde{P}_{04}^{-0.70}$ yr before encroached upon by the surrounding gas.

To determine the fraction $f_{\rm SNR}$ of the volume of the galactic disk occupied by hot SNRs which occur locally at

[1] Since both the topic under discussion and the nomenclature used to discuss it are complex, we include a glossary of symbols in Appendix A.

[2] Note that the external pressure, which has not always been included in the calculations of previous workers, will tend to stop expansion before cooling is reached if P_0 is too large: $\tilde{P}_{04} > 10^{2.23}E_{51}^{0.13}n_0^{1.23}$. Some overshooting beyond R_E is likely, and the SNR radius will subsequently oscillate about R_E.

a rate $S(\sim 10^{-13.0}\ \mathrm{pc}^{-3}\ \mathrm{yr}^{-1})$, define $dQ(R)$ as the probability that an arbitrary point in the disk is inside a SNR with a radius between R and $R + dR$. Then if $R \propto t^{\eta}$, one can show that

$$Q(R) = (1 + 3\eta)^{-1} SVt \tag{1}$$

is the expected number of SNRs which will occur within volume V having radius $R \leq R(t)$ and that the filling factor is $f = 1 - \exp(-Q)$. The value of f_{SNR} can be estimated by taking $t = t_{\max}$, $R = R_E$, and $\eta = 0$ since in its final phase the SNR does not expand; we find

$$Q_{\mathrm{SNR}} = 10^{-0.29} E_{51}{}^{1.28} S_{-13} n_0{}^{-0.14} \tilde{P}_{04}{}^{-1.30}\,, \qquad f_{\mathrm{SNR}} = 1 - \exp(-Q_{\mathrm{SNR}})\,. \tag{2}$$

Although this result was derived for a uniform medium, it should be approximately valid for the actual interstellar medium which contains embedded clouds to the extent that losses onto the clouds are not dominant. If the intercloud temperature and density were small $(T_0, n_0) \leq (10^4, 10^{-0.5})$, then the pressure would be small; and we see from equation (2) that $Q_{\mathrm{SNR}} \geq 1$ and the low-density remnants would overlap before they could be dissipated.

The conclusion is clear and rather surprising. *The standard two-phase model proposed by Field, Goldsmith, and Habing (1969) for which* $(T_0, n_0) \sim (10^4\ \mathrm{K}, 10^{-1}\ \mathrm{cm}^{-1})$ *cannot be maintained if our estimate of supernova rate S is reasonably accurate.* It would rapidly self-destruct (time $< 10^7$ years) as the relatively cool intercloud medium was swept up into dense shells and replaced by hot, low-density shock-heated gas. Thus we must consider anew the steady state energy, mass, and pressure balance in the interstellar medium which we see, by the above argument, is dominated by the pervasive hot ionized intercloud medium (HIM). We must and do find space for widespread warm components ($T \sim 10^2$–10^4 K) seen both as ionized gas (diffuse Hα, pulsar dispersion measure) and, in addition, as a largely neutral (21 cm emission) gas component.

In this paper we wish to present a comprehensive model of the interstellar medium dominated and regulated by supernova explosions. We are guided throughout by the principle stressed by Spitzer (1956) that all phases of the medium must be kept in rough pressure equilibrium. Cox and Smith (1974), Smith (1977), and Shapiro and Field (1976) have discussed some aspects of the steady state interstellar medium to be expected in the presence of a hot component. Here we shall outline a new scheme for the energy and mass balance of the interstellar medium based principally on the effects of radiative losses, evaporation of cloud material by the hot gas, and the replenishment of the high-density phase in dense shells about old SNRs.

In summary, the model developed has three essential components. Most of the space is filled with a hot low-density medium (the HIM) with typical values of the density and temperature $(n, T) = (10^{-2.5}\ \mathrm{cm}^{-3}, 10^{5.7}\ \mathrm{K})$ which is moderately inhomogeneous [probability distributions for (n, T) are given] as blast waves from supernova explosions of various ages pass by a given point; the filling factor for this component f_{HIM} is 0.7 to 0.8. Embedded in the hot medium are cold, neutral, relatively dense clouds (CNM); the filling factor f_{CNM} is 0.02–0.04, and the internal density and temperature are $(10^{1.6}\ \mathrm{cm}^{-3}, 10^{1.9}\ \mathrm{K})$. Surrounding each cloud is a warm ($T \sim 8{,}000$ K) photoionized cloud corona. This component occupies a much larger volume than the cold clouds—the filling factor being ~ 0.2—but contains far less mass. In our simplified treatment we subdivide the cloud coronae into two regions, an outer one wherein the fractional ionization is ~ 0.7 maintained by hot (B) stars, which we designate the warm ionized medium (WIM), and an inner layer of smaller volume which is nearly neutral having fractional ionization ~ 0.1 maintained by the very soft X-rays ($h\nu \sim 60$ eV) emitted by supernova remnants, which we designate WNM.

All components are in rough pressure equilibrium; and the interchange of material between the phases due to the processes of cloud evaporation, photoionization, thermal instabilities, and hydrodynamic shocks is quite rapid, with the mass in a given volume element typically changing phase in a time less than 10^6 years. Molecular cloud complexes, ordinary Strömgren spheres, and other phenomena occurring in regions of active star formation are not treated in this paper.

We analyze in § II the evolution of SNRs in a multicomponent medium, including evaporation. We find that in the early, adiabatic, phases $R \propto t^{3/5}$ rather than the $t^{2/5}$ found for the standard Sedov or conduction-modified Sedov solutions. Cloud evaporation, which is of greatest importance in the early part of the adiabatic phase, increases the density of the SNR interior to a value substantially over the ambient density. Ultimately cooling occurs at $R = R_c$, and then evolution proceeds under the combined influences of (*a*) the differences between the internal and external pressures, (*b*) the momentum stored in the shell at R_c, and (*c*) the disruptive effects of collisions between the expanding shell and interstellar clouds. Under a variety of circumstances the evolution can still be characterized by $R \propto t^{\eta'}$ with $\eta' = 2/7$, close to the value $\eta' = 0.31$ found by Chevalier (1974) for a particular case. The role of evaporation in regulating the conditions in the average hot intercloud medium is discussed in § III, where the above mentioned physical balance conditions are used to determine the properties of the average pressure, density, and temperature. The conditions necessary for a galactic wind are also determined. In § IV we introduce the warm interstellar medium composed of the radiatively ionized cloud edges; in § V we solve for the pressure and density of the HIM and the characteristics of the cold cloud distribution; and in § VI we show that our model permits significantly improved agreement with observation over previous work.

II. SNR EVOLUTION

a) Early Phases

The dynamics of SNRs are controlled by the lowest density component of the ambient medium which has a significant filling factor (Cox and Smith 1974; McKee and Cowie 1975). Let us now consider the evolution of a SNR in a uniform medium of density $n_0 \approx 10^{-2}$ cm^{-3}. The several well-defined phases of SNR evolution (cf. Woltjer 1972) will still exist in modified form.

Thermal conduction has two effects. The first, analyzed by Chevalier (1975) and Solinger, Rappaport, and Buff (1975), is that the interior temperature is approximately uniform and equal to the mean temperature in the remnant,

$$T_h = \frac{2\mu E_{\rm th}}{3kM} = 10^{10.08} R^{-3} E_{51} n_h^{-1}\ {\rm K}\,, \tag{3}$$

where μ is the mean molecular weight (we assume $n_{\rm He}/n_{\rm H} = 0.1$), the thermal energy in the remnant, $E_{\rm th}$, is taken to be 0.71 times the total energy E, M is the total mass in the remnant, and the density in the interior n_h is approximately uniform.

The second effect of conduction is that any cold clouds and warm interstellar medium will tend to be evaporated inside the hot interior of the SNR. The evaporation of spherical clouds by hot gas has been analyzed by Cowie and McKee (1977) and McKee and Cowie (1977). Provided that the mean free path for electron-electron energy exchange in the ambient gas is less than about half the cloud radius, the evaporation rate is $\dot{M}_{\rm ev} = 10^{4.44} T_h^{5/2} a\phi$ g s^{-1} per cloud, where T_h is the ambient temperature and a is the cloud radius in parsecs. The parameter ϕ is unity in the ideal case and may be less than 1 due to magnetic fields, turbulence, etc. If the cloud radius exceeds $0.16 T_{h6}^{\,2} n_h^{-1}$ pc, radiative losses exceed the conductive heating and the cloud grows by condensation.

The primary physical assumption made in deriving these results is that the magnetic field in the hot gas is connected to that in the clouds, since the conductivity perpendicular to the field is negligible. This assumption is reasonable since there is mass exchange among the three phases of the ISM even in the absence of evaporation, so that the gas now in the HIM formerly was in clouds and it carried its field along with it. The assumption is also self-consistent, since evaporation accelerates the rate of mass exchange between the HIM and the other two phases.

To describe the dynamics of an SNR in the presence of evaporation during the early adiabatic phases, we make the following approximations: (1) the density and temperature inside the SNR are independent of position (cf. Chevalier 1975); (2) the blast wave velocity is proportional to the isothermal sound speed c_h inside the SNR, $\dot{R} \equiv v_b = \alpha c_h = \alpha(0.71E/2\pi R^3 \rho_h)^{1/2}$; and (3) the only effects of the clouds on the expansion is to alter the density n_h inside the remnant and to alter α from the value 1.68 it has for the Sedov solution (estimated in the present case to be $\alpha = 2.5$).

The evolution of the remnant is then described by the equations of energy conservation $E_{\rm th} = 1.5\rho_h c_h^2 V$ and mass conservation

$$\frac{dM}{dt} = 4\pi R^2 \rho_0 v_b + N_{\rm cl} \dot{M}_{\rm ev} V\,, \tag{4}$$

where $N_{\rm cl}$ is the number of clouds per unit volume. During the early evaporation dominated phases of the SNR evolution, the second term in equation (2) exceeds the first and one readily finds that $R \propto t^{3/5}$ instead of the usual $R \propto t^{2/5}$, provided that the conduction is unsaturated ($R > 50$ pc for $n_0 \sim 10^{-2}$). Writing $R \propto t^{\eta}$, we show in Appendix D that, in terms of t (measured in years),

$$R = 10^{-0.75} \left(\frac{\alpha}{\eta}\right)^{2/5} \left(\frac{E_{51}}{n_h}\right)^{1/5} t^{2/5}\ {\rm pc}\,, \tag{5a}$$

$$Q = \frac{10^{-14.64}}{(1 + 3\eta)} \left(\frac{\alpha}{\eta}\right)^{6/5} S_{-13} \left(\frac{E_{51}}{n_h}\right)^{3/5} t^{11/5}\,. \tag{5b}$$

The equation for Q (5b) is obtained from equations (1) and (5a). Note that the early $t^{3/5}$ dependence is due to the variation in n_h, which declines as evaporation decreases in importance. The smooth change in η and n_h/n_0 is given parametrically by

$$\eta = \frac{2}{5}\left(\frac{1 + x^{5/3}}{2/3 + x^{5/3}}\right), \qquad x \equiv 10^{-1.19} R \Sigma^{1/5} n_0^{3/5} E_{51}^{-2/5}\,, \qquad n_h/n_0 = 1 + x^{-5/3}\,. \tag{6}$$

Here the constant

$$\Sigma \equiv \alpha a^2/(3 f_{\rm cl}\phi) \tag{7}$$

measures the effectiveness of evaporation; n_h/n_0 is large when x and Σ are small. The quantity $f_{\rm cl}$ is the filling factor of the cold clouds and warm medium. At $x = 1$ the density of evaporated gas equals the ambient density. The solution given by equations (5), (6), and (7) goes smoothly from the evaporation-dominated ($\eta = \frac{3}{5}$) case for small x to the (conduction modified) Sedov solution ($\eta = \frac{2}{5}$) for $x \gg 1$. During the first phase the energy expended in evaporating clouds, $\Delta E_{\rm ev} \equiv \int \dot{M}_{\rm ev}(3c_h^2/2)dt$, is quite generally $\geq E_{\rm th}$; this is not a "loss" but a dilution of the thermal energy. The mass evaporated from clouds is approximately

$$\Delta M_{\rm ev} = 10^{2.73} E_{51}^{6/5} \Sigma^{-3/5} n_h^{-4/5}\, M_\odot\,. \tag{8}$$

The solutions (5) and (6) break down when cooling sets in or when $Q > 1$, so that the SNRs overlap.

Radiative losses can occur in two ways: directly, by radiation from the hot gas, or indirectly, by radiation from isothermal shocks which are driven into the clouds. Cox and Smith (1974) focused on the latter; we shall refer to it as "cloud crushing" and concentrate here on the direct radiation losses, deferring a discussion of cloud crushing to a future publication. We define the rate at which energy is dissipated by radiation to be $\beta n_h^2 \Lambda(T_h) V$, where β is the enhancement of the cooling rate due to internal variations of temperature and density. For the Sedov solution, the density variation gives $\beta = 2.3$ (Cox 1972). An equally important effect is the enhanced radiation from the conductive interfaces between the clouds and hot gas (McKee and Cowie 1977), which depends on the temperature T_h and the cloud filling factor $f_{\rm cl}$; altogether, we estimate $\beta = 10$. Using $\Lambda = 6.2 \times 10^{-19} T^{-0.6}$ for 10^5 K $< T < 4 \times 10^7$ K (inferred from Raymond, Cox, and Smith 1976), we expect (following Chevalier 1974) that half the SNR energy has been radiated and a dense shell forms when the cooling time equals the age. For reference we collect the values of various parameters at the cooling point defined in this manner, based on equations (5) and (6) with $\eta = \frac{3}{5}$:

$$\begin{aligned}
R_c &= 10^{2.21} E_{51}^{0.04} \alpha^{0.19} \beta^{0.04} (Q_c/S_{-13})^{0.23} = 10^{2.26}\ \text{pc}\,,\\
t_c &= 10^{6.18} E_{51}^{-0.11} \alpha^{-0.20} \beta^{-0.11} (Q_c/S_{-13})^{0.31} = 10^{5.90}\ \text{yr}\,,\\
v_{bc} &= 10^{1.80} E_{51}^{0.15} \alpha^{0.39} \beta^{0.15} (Q_c/S_{-13})^{-0.08} = 10^{2.13}\ \text{km s}^{-1}\,,\\
n_{hc} &= 10^{-2.04} E_{51}^{0.59} \alpha^{-0.14} \beta^{-0.41} (Q_c/S_{-13})^{-0.54} = 10^{-2.34}\ \text{cm}^{-3}\,,\\
\tilde{P}_{hc} &= 10^{3.80} E_{51}^{0.89} \alpha^{-0.58} \beta^{-0.11} (Q_c/S_{-13})^{-0.69} = 10^{3.67}\ \text{K cm}^{-3}\,,\\
T_{hc} &= 10^{5.47} E_{51}^{0.30} \alpha^{-0.45} \beta^{+0.30} (Q_c/S_{-13})^{-0.15} = 10^{5.64}\ \text{K}\,,
\end{aligned} \tag{9}$$

the numerical values in the last column being taken from § IV. Expressions for R, t, etc., at other values of Q are given in Appendix B. Note that, because of evaporation, n_{hc} exceeds the ambient density n_0. Development of the dense shell at $R = R_c$ occurs at a thermally unstable part of the cooling curve and proceeds rapidly. If a fraction δ of the interior mass collects in the dense shell, then the density and pressure just after dense shell formation are $(\rho_{hc}', P_{hc}') = (1 - \delta) \times (\rho_{hc}, P_{hc})$ and the shell mass is initially $M_{s0} = \delta \times (4\pi/3)\mu_{\rm H} R_c^3 n_{hc}$, where $\mu_{\rm H}$ is the mass per hydrogen nucleus.

b) *Late Phases*

During the late stages of evolution ($R > 100$ pc) several competing complicating effects must be considered; we can quantitatively assess the importance of these effects by noting five length scales.

1. *Cooling.*—The radius R_c is given by equation (9).

2. *Anisotropy.*—Let the scale height of the ambient medium be H; then if $R_c/H \gg 1$, the SNR will blow out of the galaxy (*cf.* Chevalier and Gardner 1974), releasing most of its energy into the halo.

3. *External pressure.*—If $\tilde{P}_{04}$ is sufficiently large, the supernova will come into equilibrium with the interstellar pressure at radius $R_E = 10^{2.15}(E_{51}/\tilde{P}_{04})^{1/3}$ without dense shell formation. This occurs if $R_c/R_E > 1$.

4. *Multiple events.*—If the supernova rate is large enough, overlap will occur before cooling and dense shell formation. Solving equation (5) for $R(Q = 1) \equiv R_{ov}$, we find $R_{ov} = 10^{1.91}[\alpha(3\eta + 1)/\eta S_{-13}]^{2/11}(E_{51}/n_h)^{1/11} < R_c$ is the condition.

5. *Cloud-shell collisions.*—Dense shells have a smaller mass per unit area than standard clouds. Thus holes will be formed temporarily each time the shell passes a cloud, and the maximum surface density that can be accumulated between successive passages of clouds through a particular part of the expanding front is $n_0\lambda_w$, where λ_w is the typical distance between cold clouds along a given line of sight $\lambda_w \equiv (N_{\rm cl}\pi a^2)^{-1}$. Cloud evaporation at these late phases is unimportant. If the shock velocity is sufficiently high, this column density will be inadequate for cooling and the dense shell cannot reform. The column density required for the postshock temperature to drop by a factor 2 is $\mathcal{N}_{\rm cool} \simeq 3 \times 10^{17} v_7^{4.2}$ cm^{-2} for $v_7 > 1$, whereas we estimate below that $n_0\lambda_w \approx 5 \times 10^{16}$ cm^{-2}. Thus condensation of cold matter will typically occur only on clouds passed by shocks during late stages of SNR expansion.

Unfortunately, all the length scales are comparable, being within 50% of 100 pc. If our estimate for the filling factor of the clouds (CNM + WIM + WNM) derived later is correct, then λ_w is the smallest length. The effect of cloud-shell collisions becomes important only for $R > R_c$, and because the maximum SNR radius $R_{ov} \sim R_c + \lambda_w$, cloud-shell collisions do not have space to dominate the dynamics of the SNR (see Cowie, McKee, and Ostriker 1977 for a more detailed discussion) even though λ_w is small.

The mass of the swept-up matter is

$$M_s = \delta\left(\Delta M_{\rm ev} + \frac{4\pi}{3}\mu_{\rm H} n_0 R_c^3\right) + \frac{4\pi}{3}\mu_{\rm H} n_0 (R^3 - R_c^3)\,, \tag{10}$$

where the evaporated mass $\Delta M_{\rm ev}$ is assumed to increase no longer for $R > R_c$.

The expansion of the shell is governed by

$$\frac{d(M_s v_b)}{dt} = 4\pi R^2\left[P_{hc}'\left(\frac{R_c}{R}\right)^5 - P_0\right], \tag{11}$$

where P_{hc}' is the interior pressure just after cooling. First consider the case in which $\Delta M_{\rm ev}$ is not significant and $M_s \approx (4\pi/3)\rho_0 R^3$; then if the ambient pressure P_0 is negligible, this leads to the pressure-modified Oort (1951) solution

$$R = 10^{-0.32}\left(\frac{R_c^2 E_{51}}{n_0}\right)^{1/7} t^{2/7}\,, \tag{12}$$

which agrees quite well with Chevalier's (1974) numerical solution.[3] Alternatively, if the evaporated mass is dominant ($M_s \approx \delta\Delta M_{\rm ev} =$ const.), then one finds $v_b \sim$ const. In fact, we shall find that the evaporated mass is comparable to the swept-up mass, so that the actual dynamics for SNRs in the dense shell phase lie somewhere between these extremes. In particular, if we write $R \propto t^{\eta'}$ after cooling, the parameter η' is known to an accuracy of only $2/7 < \eta' < 1$.

III. EQUILIBRIUM CONDITIONS, MAINTENANCE OF A HOT MEDIUM

Let us now turn to the cumulative effect of many supernova remnants on the ISM. We shall assume that direct radiation dominates over cloud crushing ($\int PdV$) losses, which can be shown to be valid of $f_{\rm cl} < \frac{1}{3}$, and that we can consider the ISM to be in a steady state on time scales of 10^6 years. Equilibrium is determined by pressure, mass, and energy balance.

a) Pressure Balance

The gas in the interior of an old SNR expands until it reaches pressure equilibrium with the ambient medium. Since there is little further energy loss after SNRs collide, the ambient pressure must be that inside a remnant at the moment of overlap ($Q = 1$). After cooling, we shall take $R \propto t^{\eta'}$, so that at overlap

$$R_{ov} = R_c Q_c^{-\eta'/(3\eta'+1)}\,. \tag{13}$$

The gas in the interior expands nearly adiabatically in this phase ($R_c < R < R_{ov}$), so the pressure at $Q = 1$ is

$$P_0 = P_{hc}' Q_c^{5\eta'/(3\eta'+1)}\,. \tag{14}$$

b) Mass Balance

After radiative cooling sets in, an expanding SNR sweeps up the HIM it encounters into a dense shell which, as noted earlier, is deposited rapidly onto clouds as these are passed. Addition of already cold clouds does not affect the net balance, and further cloud evaporation is small. Then, if about half of the mass inside the SNR at the cooling point goes into the shell ($\delta = \frac{1}{2}$), equating the mass added to the HIM by evaporation to that lost by shell formation[4] gives

$$(n_{hc} - n_0)R_c^3 = \tfrac{1}{2} n_{hc} R_c^3 + n_0 (R_{ov}^3 - R_c^3)\,, \tag{15}$$

since mass sweeping continues until the shocks collide at $R = R_{ov}$. This is valid even if $\lambda_w < (R_{ov} - R_c)$; swept-up

[3] We note in passing the remarkable fact that in the opposite limit where cloud-shell collisions dominate, then for $(R - R_c) \gg \lambda_w$, the $R(t)$ behavior is exactly as given by eq. (12) except that the numerical coefficient is smaller by $6^{1/7} = 1.292$.

[4] Mass balance by dense shell formation is appropriate for small values of the cloud filling factor $f_{\rm cl}$, which is the case we are considering. For sufficiently large values of $f_{\rm cl}$, condensation of the hot gas onto clouds in old SNRs may be the dominant process. The typical conditions we find below for the HIM are close to those required for condensation.

matter is deposited on clouds as the supernova shock passes them by. Rewriting equation (15) and applying (13) gives

$$n_{hc} = 2n_0\left(\frac{R_{ov}}{R_c}\right)^3 = 2n_0 Q_c^{-3\eta'/(1+3\eta')} . \tag{16}$$

Since the interior density $n_h \propto R^{-3}$ after the shell forms at R_c, this simply states that the ambient density $n_0 = n_h(Q = 1)$, the background density in the HIM, is the density in a SNR when it overlaps another of the same age. This equation cannot be satisfied except in the evaporative limit ($x_c > 1$, where $x_c \equiv x[R = R_c]$); otherwise the right-hand side of equation (16) exceeds the left-hand side and evaporation becomes more important either because the cool phase grows and Σ decreases, or because the temperature in the hot medium increases as part of it is swept up into dense shells.

c) *Energy Balance*

In a steady state the injected energy must be dissipated. Here we require that this dissipation be due to radiation from the hot gas, so that dense shell formation occurs. In the notation of § II*b* this requirement is

$$R_c < \text{Min}\,(H, R_E, R_{ov}) \approx \text{Min}\,(H, R_{ov}) , \tag{17}$$

since the pressure balance condition (§ III*a*) essentially equates R_E and R_{ov}.

d) *Results*

Given the uncertainty in the parameter η' (cf. § II*b*) we shall neglect the difference between $3\eta'/(1 + 3\eta')$ in equation (16) and 0.54 in equation (9) and take $\eta' \approx 0.39$. The ambient density is then

$$n_0 = 10^{-2.34} E_{51}{}^{0.59} \alpha^{-0.14} \beta^{-0.41} S_{-13}{}^{0.54} , \tag{18}$$

which is independent of Q_c and dependent on only a single parameter, *SE*, the rate of supernova energy input into the interstellar medium. The ambient pressure (eq. [14]) is

$$\tilde{P}_0 = 10^{3.50} E_{51}{}^{0.89} \alpha^{-0.58} \beta^{-0.11} S_{-13}{}^{0.69} Q_c{}^{0.21} . \tag{19}$$

The uncertainty in η' renders both n_0 and P_0 approximate, although the error is small for Q_c of order unity.

The mass balance condition also allows us to determine Q_c in terms of the cloud evaporation parameter Σ. This condition demands $n_{hc} \gg n_0$ so that we are in the evaporative limit and can approximate equation (6) by $n_h \approx n_0 x^{-5/3}$. Then the density in the adiabatic phase of SNR evolution is

$$n_h = 10^{1.99} E_{51}{}^{2/3} \Sigma^{-1/3} R^{-5/3}\ \text{cm}^{-3} \quad (10^{1.5} \lesssim R < R_c) . \tag{20}$$

Combining this result with equation (9) yields

$$Q_c = 10^{-2.09} E_{51}{}^{-0.09} \alpha^{1.18} \beta^{-2.19} S_{-13} \Sigma^{2.10} . \tag{21}$$

The value of Σ is restricted by the requirements given by equation (17). Satisfying that equation, rewritten as

$$\Sigma < [10^{1.17} E_{51}{}^{-0.04} (\alpha/\beta)^{-0.96},\ 10^{1.00} E_{51}{}^{0.04} \alpha^{-0.56} \beta^{1.04} S_{-13}{}^{-0.48}] , \tag{22}$$

guarantees that $R \to R_c$ in the evaporative limit, that explosions not escape the galaxy, and that $Q_c < 1$. We have taken the thickness of the galactic disk to be $H = 200$ pc. These limits are comparable, extremely insensitive to E_{51}, and, for $(\alpha, \beta) = (2.5, 10)$, give $\Sigma < 10^{1.75}$. For comparison, Spitzer's (1968) "standard clouds" give $\Sigma_{\text{CNM}} = 10^{2.77}$ if $\phi = 1$; the cold medium by itself is insufficiently pervasive to contain supernova explosions and to force them to radiate away their energy in the galactic disk. For standard values of the observational parameters $(S_{-13}, E_{51}) = (1, 1)$ we have

$$\Sigma = 10^{1.82} Q_c{}^{0.48} , \tag{23}$$

so that inequality (22) is satisfied if $Q_c < 0.7$.

The ambient hot medium for these standard parameters is

$$\begin{aligned} \text{Ambient hot medium, } Q = 1:\ n_0 &= 10^{-2.80}\ \text{cm}^{-3} , \\ \tilde{P}_0 &= 10^{3.16} Q_c{}^{0.21}\ \text{K cm}^{-3} , \\ T_0 &= 10^{5.60} Q_c{}^{0.21}\ \text{K} . \end{aligned} \tag{24}$$

These are the lowest values of these parameters in the medium. An important consequence of this model for the ISM is that there are significant deviations from pressure equilibrium. At a typical point in the HIM, the values of n, P, T are those in the vicinity of $Q = \frac{1}{2}$, so we define

$$(n_h)_{\rm typ} = 2\int_{0.25}^{0.75} n_h dQ\,, \tag{25}$$

etc. For $0.25 \leqslant Q_c \leqslant 0.75$, the resulting values are

$$\begin{aligned} \text{Typical hot medium: } (n_h)_{\rm typ} &= 10^{0.21} n_{hc} Q_c^{1.10} = 10^{-2.29} Q_c^{0.56} = 3.5 \times 10^{-3}\ {\rm cm^{-3}}\,, \\ (\tilde{P}_h)_{\rm typ} &= 10^{0.32} \tilde{P}_{hc} Q_c^{1.42} = 10^{3.78} Q_c^{0.73} = 3.6 \times 10^{3}\ {\rm cm^{-3}\ K}\,, \\ (T_h)_{\rm typ} &= 10^{0.11} T_{hc} Q_c^{0.32} = 10^{5.70} Q_c^{0.17} = 4.5 \times 10^{5}\ {\rm K}\,, \end{aligned} \tag{26}$$

where in the second column we have approximated the sum of the two terms arising from $Q < Q_c$ and $Q > Q_c$ (see Appendix B) by a single power law, in the third column we have used the standard values of the parameters S, E_{51}, α, and β, and in the final column we have set $Q_c = \frac{1}{2}$ (see § V). By comparison, the average values[5] $\langle f \rangle \equiv \int_0^1 f(Q) dQ$ tend to be dominated by adiabatic SNRs and can be significantly higher:

$$\begin{aligned} \text{Average hot medium: } \langle n_h \rangle &= 10^{0.19} n_{hc} Q_c^{0.63} = 10^{-2.31} Q_c^{0.09} = 4.6 \times 10^{-3}\ {\rm cm^{-3}}\,, \\ \langle \tilde{P}_h \rangle &= 10^{0.45} \tilde{P}_{hc} Q_c^{0.84} = 10^{3.94} Q_c^{0.15} = 7.9 \times 10^{3}\ {\rm K\ cm^{-3}}\,, \\ \langle T_h \rangle &= 10^{0.15} T_{hc} Q_c^{0.40} = 10^{5.74} Q_c^{0.25} = 4.6 \times 10^{5}\ {\rm K}\,. \end{aligned} \tag{27}$$

We see that the temperatures defined by different averages (24), (26), (27) differ only slightly but the pressures differ substantially.

We note here that our treatment of the background medium in §§ II and III is consistent. New supernovae always tend to expand into the lowest-density, lowest-pressure medium available so that the ambient conditions used in the discussion of SNR evolution in § II are approximately the same as the ambient conditions defined in § III at overlap, $n_0 = n_h(Q = 1)$ and $P_0 = P(Q = 1)$.

In §§ IV and V we shall determine Σ and Q_c self-consistently from an analysis of the equilibrium distribution of the sizes of the clouds. Observationally, the situation is uncertain, but in § V we find $Q_c = 0.5$.

e) The Gaseous Galactic Halo

It is easy to see that, when the present model is applied to the galactic halo in a manner analogous to Mathews and Baker (1971), one naturally finds a gaseous halo of the type envisioned by Spitzer (1956) in his classical study. The pressure scale height of the hot gas in the disk is $h = 5T_6$ kpc for a surface stellar mass density of 100 $M_\odot$ pc^{-2}, so we expect hot gas at the ambient pressure $\tilde{P}_0 = 1250$ K cm^{-3} to extend many kiloparsecs above the plane. The low density of clouds in the halo prevents evaporative regulation and the associated radiative losses, so the halo temperature should be somewhat higher than in the disk, $T_{\rm halo} \gtrsim 10^6$ K, and the density significantly lower, $n_{\rm halo} \lesssim 10^{-3.3}$ cm^{-3}. Energy is supplied to the halo by supernovae in the disk and in the halo. The energy supply from the disk is very roughly $2\pi(10\ {\rm kpc})^2 P_0 c_0 \approx 10^{40}$ ergs s^{-1}. Supernovae are known to occur well outside the disk of the galaxy; for example, Kepler's supernova is about 1 kpc above the plane. Very few radio remnants are observed at such a height, but that could be because an ambient density in excess of 10^{-3} cm^{-3} is required to produce detectable radio emission. If we estimate a halo supernova rate which is 30 times less than the disk rate (and therefore comparable to that in elliptical galaxies [Tammann 1974]), the resulting energy supply to the halo is $\dot{U} = 10^{40.5} E_{51}$ ergs s^{-1}. Essentially all this energy is available for heating the halo because, at the low ambient density, the SNRs come into pressure equilibrium with the ambient medium (at $R_E = 280$ pc) before they cool ($R_c = 430$ pc).

Energy losses are due to radiation and to a galactic wind (Mathews and Baker 1971). A halo of radius 15 kpc and thickness $2h$ radiates $10^{40.0} T_6^{-1.6}$ ergs s^{-1}. For $T_6 \gtrsim 1$ this cannot balance the heating due to halo SNRs, so a wind must occur. (For $T_6 \sim 0.5$, radiative losses would dominate and a "galactic fountain" [Shapiro and Field

[5] An exact value for the average of a quantity in the HIM is difficult to calculate because of interactions between different SNRs. Ignoring the interactions gives $\langle f_{\rm no\,int} \rangle \equiv \int_0^\infty f(Q) \exp(-Q) dQ = f_1 \Gamma(n + 1)$ for $f = f_1 Q^n$. The average we adopt,

$$\langle f \rangle \equiv \int_0^1 f(Q) dQ = f_1/(n + 1)\,,$$

crudely includes the interactions by assuming they all occur at $Q = 1$. The pressure, density, and temperature all decrease with $Q(n < 0)$ and have $\langle f \rangle > \langle f_{\rm no\,int} \rangle$, as expected.

1976] would result). If the sonic point (denoted by the subscript s) occurs at one pressure scale height above the plane, then the mass loss rate is $\dot{m} = 2\pi r_s^2 \rho_s v_s = 2\pi r_s^2 (P_0/e) c_s^{-1} = 10^{25.87} T_{s6}^{-1/2}$ g s^{-1} for a halo radius at the sonic point of 15 kpc. The energy balance condition $\dot{U} = 3\dot{m} c_s^2$ then gives $T_{s6}^{1/2} = 3.4 \dot{U}_{41} \approx 1$ and $\dot{m} \approx 1\ M_\odot$ yr^{-1}. The mass loss rate would be lower if the sonic point occurred farther out in the flow and/or if T_s were higher; mass loss from halo stars alone could make up about 0.1 $M_\odot$ yr^{-1} (Ostriker and Thuan 1975). At a temperature of 10^6 K, the halo density would be 5.4×10^{-4} cm^{-3}, which will produce detectable X-ray emission (both continuum and lines), relatively narrow O VI absorption lines from the quiescent halo gas, and optical emission in the coronal lines, although the small emission measure ($n_e^2 l = 2.1 \times 10^{-3} T_6^{-1}$ cm^{-6} pc at the pole) would require a specially designed experiment.

IV. A SPECTRUM OF CLOUDS WITH IONIZED EDGES

In the model developed here the warm ($T \sim 10^4$ K) medium does not pervade most of space. Rather, it is confined to small clouds and envelopes surrounding cold clouds, and it exists as the natural consequence of immersing the latter in the UV (13 eV $< h\nu <$ 40 eV) and soft X-ray (40 eV $< h\nu <$ 120 eV) radiation emitted by stars and supernova remnants. Most of this warm medium will be ionized, as we show below. Here we shall explore the implications of assuming a spectrum of cloud sizes. Our treatment will be largely independent of the analysis of §§ I–III; combination of the results of this section with the previous ones yields a self-consistent description of the ISM, as shown in § V.

Hobbs's (1974) observations of cold clouds show that for equivalent hydrogen column densities in the range $10^{20.2}$ cm$^{-2} < \mathcal{N} < 10^{21.2}$ cm^{-2} the number of clouds per unit column density varies as $\mathcal{N}^{-2}$. If the densities in the clouds are approximately constant, and the clouds are spherical with radius a, then the number per unit volume having radii $a \to a + da$ is proportional to a^{-4}. Small clouds have a large fraction of their volume in the low-density warm medium. Thus we define a_0 as the radius the cloud would have in the absence of ionizing radiation (if it were entirely cold), and adopt for explicitness the power law distribution for number of clouds per unit volume with a_0 in the range $a_0 \to a_0 + da_0$:

$$N(a_0) da_0 = N_0 a_0^{-4} da_0\,, \qquad a_{0l} < a_0 < a_{0u}\,, \qquad N_0 = 3N_{\rm cl} a_{0l}^3 [1 - (a_{0l}/a_{0u})^3]^{-1} \doteq 3N_{\rm cl} a_{0l}^3\,, \tag{28}$$

where the maximum radius a_{0u} is determined by the instability of clouds which are too large to gravitational collapse and the lower limit a_{0l} is determined by the efficient destruction of clouds having low surface density by various processes; $N_{\rm cl}$ is the spatial density of clouds (pc^{-3}). One further relevant length scale a_{0b} exists. Large clouds are enclosed in radiatively ionized shells with inner and outer radii (a_c, a_w), but those smaller (less massive) than a certain size a_{0b} are optically thin; they would have no cold cores, the value of a_{0b} depending primarily on the ionizing background. The physical size of clouds at this break point is

$$a_{wb} = (n_c/n_w)^{1/3} a_{0b}\,. \tag{29}$$

Since clouds without cold cores will be evaporated even in the relatively cool ambient medium between successive SNR passages and also have such a low mean surface density that they are easily swept up onto shocks from supernovae in the postcooling phase, the cloud spectrum will terminate when the cold cores are comparable in mass to the tenuous warm coronae. Thus we write

$$a_{0l}^3 = k_l a_{0b}^3\,, \tag{30}$$

where the constant k_l is assumed to lie in the range $1 \leq k_l \ll n_c/n_w$; if the cold and warm masses of the smallest clouds are equal, then $k_l = 2$. As is shown in Appendix C, the filling factors for the cold and warm media are then, to lowest order in (n_w/n_c) and $(a_{0l}/a_{0u})^3$,

$$f_c = 4\pi a_{wb}^3 N_{\rm cl} k_l \left(\frac{n_w}{n_c}\right) \ln (a_{0u}/a_{0l})\,, \qquad f_w = \tfrac{4}{3}\pi a_{wb}^3 N_{\rm cl}\,; \tag{31}$$

and the intercloud distances are $\lambda^{-1} \equiv \int N(a_0) \pi a^2 da$,

$$\lambda_c = [2.172 k_l \pi N_{\rm cl} a_{wb}^2 (n_w/n_c)^{2/3}]^{-1}\,, \qquad \lambda_w = (\pi N_{\rm cl} a_{wb}^2)^{-1} \tag{32}$$

in the same approximation. The *ratios* of the filling factors are fixed largely by our geometrical modeling of the interstellar medium and the requirement of pressure balance:

$$f_c/f_w = 3k_l (n_w/n_c) \ln (a_{0u}/a_{0l}) = 0.14\,, \qquad \lambda_c/\lambda_w = 0.46 (n_c/n_w)^{2/3} k_l^{-1} = 6.8\,, \tag{33}$$

the numerical values being taken from the next section. One further moment is needed: the evaporation parameter Σ defined by equation (7) is

$$\Sigma = \left(\frac{8\pi}{15} N_{\rm cl} a_{wb}\right)^{-1} \tag{34}$$

for $\alpha = 2.5$ and an assumed efficiency $\phi = \frac{1}{3}$.

To evaluate the filling factors, etc., for the interstellar medium, we relate the cloud properties to the mean density $\bar{n}$ in the ISM and to the production rate of ionizing photons per unit volume ϵ_{UV}. We also impose the condition of pressure equilibrium between the WIM and the CIM:

$$n_c T_c = (1 + x_w) n_w T_w = 10^{3.96} \tilde{P}_4 , \tag{35}$$

where $x_w = n_e/n_w$ is the fractional ionization in the WIM and where the partial pressure due to helium has been taken as 10% of that due to hydrogen. Below we shall find that the characteristic WIM radius a_{wb} is of order 2 pc, so the WIM responds to small changes in the pressure in $\sim 2 \times 10^5$ yr. Now the mean interval between SNRs smaller than R is

$$\Delta t = \left(\frac{4\pi}{3} R^3 S\right)^{-1} = \frac{t}{(3\eta + 1)Q} = 4 \times 10^5 \text{ yr} \tag{36}$$

for $R \sim R_c$ and $Q_c \sim \frac{1}{2}$. Thus the WIM will be approximately in pressure equilibrium with the local HIM. The sound travel time across the smallest clouds, which have $a_{0l} \sim 0.5$ pc (see below) and are most numerous, is about 5×10^5 yr. Such small clouds can remain in rough pressure equilibrium with the WIM, but larger cold clouds cannot. Since we shall be interested primarily in averages over the ISM, the effect of these deviations from pressure equilibrium should not affect our results too strongly.

With the assumption of pressure equilibrium, the relation between the minimum cloud size a_{0l} and the critical radius a_{wb} can be expressed as

$$a_{0l} = \left[\frac{k_l T_c}{(1 + x_w) T_w}\right]^{1/3} a_{wb} . \tag{37}$$

The maximum cloud size $a_{0u} \approx a_{cu}$ is determined by the condition that the cloud be stable against gravitational collapse. In the absence of a magnetic field, Spitzer's (1968) analysis gives a maximum radius $2.29(T_c/80\ \text{K})\tilde{P}_{04}^{-1/2}$ pc, for which largest stable cloud the mean interior pressure is 4 times the ambient value. If one treats the cloud spectrum $N(a_0)$ as a *mass* spectrum, this corresponds to a value of a_0 which is $4^{1/3}$ times larger, or $a_{0u} = 3.63(T_c/80)P_{04}^{-1/2}$ pc ~ 6 pc for the values found below. The magnetic field in the cloud exerts a stabilizing influence. Following Mouschovias (1976) we take $B = 3 \times 10^{-6} n^{\kappa}$ gauss, where $\frac{1}{3} \lesssim \kappa \lesssim \frac{1}{2}$; the fact that $\kappa < \frac{2}{3}$ allows for nonhomologous contraction of the cloud along the field lines. Generalizing Spitzer's analysis to this case, we find $a_{0u} = 11.5 P_{04}^{-5/6}$ pc. Because the cloud must withstand SNR shocks from time to time, the average pressure is appropriate. We shall take a_{0u} to be 10 pc with an estimated uncertainty of 20%.

The number density of clouds N_{cl} can be related to the mean density $\bar{n} = n_c f_c + n_w f_w \approx n_c f_c$ (the warm component contributes only about 5% of the total) by using equations (31) and (35)

$$\bar{n} = \frac{10^{1.06} k_l a_{wb}{}^3 N_{cl} \tilde{P}_4}{(1 + x_w) T_{w4}} \ln (a_{0u}/a_{0l}) . \tag{38}$$

We can now determine x_w and a_{wb} from considerations of ionization balance. Let ζ be the primary photoionization rate of hydrogen per hydrogen atom. In our discussion of UV ionization we neglect helium; then, since secondary ionizations are negligible at UV energies, ζ is also the total ionization rate per hydrogen atom. Define J as the mean intensity of UV ionizing photons in the ISM (photons cm^{-2} s^{-1} sr^{-1}). At a typical point in the corona of a cloud, ionizing photons are incident from 2π steradians so that

$$\zeta = 2\pi J \sigma_H , \tag{39}$$

where σ_H is the effective cross section for photoionization of hydrogen. Then in each optically thick cloud the ionization rate balances the rate of incoming photons:

$$4\pi^2 a_w{}^2 J = \tfrac{4}{3}\pi(a_w{}^3 - a_c{}^3)\zeta n_w (1 - x_w) . \tag{40}$$

The smallest optically thick cloud has $a_c = 0$ and $a = a_{wb}$ by definition, which gives

$$a_{wb} = \frac{3}{2\sigma_H n_w (1 - x_w)} \tag{41}$$

and

$$a_w{}^2 a_{wb} = a_w{}^3 - a_c{}^3 , \tag{42}$$

an equation used repeatedly in Appendix C to determine the average cloud properties.

The equation of ionization equilibrium is (43)

$$\alpha_{\rm rec} n_e^2 = \zeta n_w (1 - x_w) ,$$

where the recombination coefficient is $\alpha_{\rm rec} = 10^{-12.59} T_{w4}^{-0.80}$. Since the recombination time $(\alpha_{\rm rec} n_e)^{-1} = 10^{5.09} n_e^{-1}$ yr is shorter than but comparable to the mean interval between SNRs, the assumption of ionization equilibrium is only approximate.

Finally, we cast our results in terms of the mean production rate of ionizing UV photons $\epsilon_{\rm UV}$:

$$\epsilon_{\rm UV} = 4\pi^2 J \langle a_w^2 \rangle N_{\rm cl} = 4\pi^2 J a_{wb}^2 N_{\rm cl} . \tag{44}$$

Combining this with equations (41) and (43), we obtain the ionization equilibrium equation in the form

$$\alpha_{\rm rec} n_e^2 \frac{4\pi}{3} N_{\rm cl} a_{wb}^3 = \alpha_{\rm rec} n_e^2 f_w = \epsilon_{\rm UV} . \tag{45}$$

Equations (35), (37), (38), (41), and (45) determine the density and ionization of the WIM and the cloud parameters a_{0l}, a_{wb}, and $N_{\rm cl}$. On both theoretical and observational grounds (see below) we have assumed that the cloud distribution extends down to $a_{0l} \sim a_{0b}$. Thus most clouds have the same outer radius $a_w \sim a_{wb} \sim 2$ pc and a core radius $a_c \sim a_{0b} \ll a_{wb}$. Some of the results of this section could have been obtained by assuming a "standard cloud"; however, for the adopted distribution $N \propto a_0^{-4}$, a significant fraction of the mass of the CNM is contained in the rare, massive clouds.

V. APPLICATION TO THE ISM

In the preceding sections we have developed approximate analytic theories of the interaction of SNRs with the ISM and of the cloud structure of the ISM. The main observational data required are the supernova rate S per unit volume, the energy E per supernova, the mean density $\bar{n}$ of the ISM, and the mean production rate $\epsilon_{\rm UV}$ of UV photons. The results also depend on the temperatures T_c and T_w, which are obtainable in principle from the energy balance of the CNM and WIM. The parameters of the cloud distribution which are needed are the index of the power law, taken to be 4 from Hobbs's (1974) observations, the maximum cloud radius a_{0u}, estimated to be 10 pc, and the minimum cloud radius a_{0l}. Finally, there are the theoretical parameters $\alpha \sim 2.5$, $\beta \sim 10$, $\phi \sim \frac{1}{3}$, $\eta' \sim 0.39$, and the fraction of the SNR mass which goes into the shell, $\delta \sim \frac{1}{2}$. In view of the many approximations we have made, close agreement with observation is not expected; nonetheless, we show that our general but highly approximate theoretical approach, with reasonable choices of the above parameters, leads to a model of the ISM consistent with numerous observations.

a) *WIM and CNM*

The density of the gas in the ISM depends on the pressure, which obeys the probability distribution evaluated in § III. Here we first determine the conditions at the typical pressure (an average over $0.25 \leq Q \leq 0.75$ [see eq. (26)]) and then determine the global values of $\langle n_e \rangle_{\rm ISM}$ and $\langle n_e^2 \rangle_{\rm ISM}$, which involve averaging over the entire ISM.

As discussed in § II, the effectiveness of the evaporation depends upon the quantity Σ defined in equation (7) and given for the assumed cloud spectrum by equation (34) (because most clouds have radii $\sim a_{wb}$). Matching this calculated value to the value of Σ required in a steady state (equation 23) and using equation (38) to eliminate $N_{\rm cl} a_{wb}^2$ gives

$$a_{wb}^2 = \frac{10^{1.21} \bar{n} T_{w4} (1 + x_w)}{k_l Q_c^{0.25} \ln (a_{0u}/a_{0l})} , \tag{46}$$

where we have set $\tilde{P} = \tilde{P}_{\rm typ} = 10^{3.78} Q_c^{0.73}$ (eq. [26]). Combining this with equation (41) gives an equation for x_w:

$$\frac{1 - x_w}{(1 + x_w)^{1/2}} = \frac{10^{-0.66}}{Q_c^{0.60} \sigma_{-18}} \left(\frac{k_l T_{w4}}{\bar{n}} \ln a_{0u}/a_{0l} \right)^{1/2} , \tag{47}$$

where $\sigma_{-18} = \sigma_{\rm H}/10^{-18}$ cm^2. Also, eliminating $N_{\rm cl} a_{wb}^3$ from equation (38) by equation (45) yields

$$Q_c^{0.73} = \frac{10^{-1.68} \epsilon_{-15} k_l T_{w4}^{1.80} (1 + x_w) \ln (a_{0u}/a_{0l})}{\bar{n} x_w^2} , \tag{48}$$

where $\epsilon_{-15} = \epsilon_{\rm UV}/(10^{-15}$ photons cm^{-3} s^{-1}).

We can now solve equations (37), (46), (47), and (48) for a_{0l}, a_{wb}, x_w, and Q_c given input values of k_l, T_c, T_w, a_{0u}, $\bar{n}$, and ϵ_{-15}. We adopt $T_c = 80$ K, $T_w = 8000$ K, and $\bar{n} = 1$ cm^{-3} with an uncertainty of perhaps 20% in each quantity (Spitzer 1977). Because clouds without cores are readily destroyed in SNR shocks, we assume that the smallest clouds have cores equal in mass to their ionized envelopes, which corresponds to $k_l = 2$. We take $\sigma_H = 3 \times 10^{-18}$ cm^2 for ionizing photons with a spectral index of 3.5 and a mean energy of 20 eV. The production rate of "free" UV photons which are not absorbed by gas near the source is poorly known. Chevalier (1974) has estimated that SNRs produce $0.35E_0$ in photons between 13.6 and 40 eV, which corresponds to $\epsilon_{UV} = 1.2 \times 10^{-15}$ photons cm^{-3} s^{-1}. Unshielded B stars give $\epsilon_{UV} = 0.7 \times 10^{-15}$ (Torres-Peimbert, Lazcano-Araujo, and Peimbert 1974; Elmergreen 1975), and planetary nebula central stars and other low-mass dying stars probable give a comparable contribution. Unshielded O stars provide up to 14×10^{-15} (Torres-Peimbert, Lazcano-Araujo, and Peimbert 1974), but this is probably absorbed by gas within 100 pc or so (Elmergreen 1976). Since some of the photons escape the galaxy altogether, we shall adopt $\epsilon_{-15} = 2.0$, recognizing an uncertainty of approximately a factor 2.

To solve the equations, we guess values for $\ln(a_{0u}/a_{0l})$, x_w, and Q_c, calculate a_{wb} from equation (46) and $\ln(a_{0u}/a_{0l})$ from equation (36) with $a_{0u} = 10$ pc; then x_w follows from equation (47) and Q_c from equation (48). The equations have been arranged so that this procedure converges rapidly. Note that $a_{wb} \approx 2$ pc for a wide choice of parameters, and that $x_w \sim 0.7$, a surprisingly low value. We find

$$\tilde{P}_{\rm typ} = 3.7 \times 10^3\,, \quad a_{wb} = 2.1 \text{ pc}\,, \quad a_{0l} = 0.48 \text{ pc}\,, \quad a_{0u} = 10 \text{ pc}\,, \quad \lambda_c = 88 \text{ pc}\,,$$
$$f_c = 0.024\,, \quad N_{\rm cl} = 5.9 \times 10^{-3} \text{ pc}^{-3}\,, \quad n_{ew} = 0.17 \text{ cm}^{-3}\,, \quad n_w = 0.25 \text{ cm}^{-3}\,,$$
$$n_c = 42 \text{ cm}^{-3}\,, \quad \lambda_w = 12 \text{ pc}\,, \quad f_w = 0.23\,, \quad Q_c = 0.51\,, \tag{49}$$

using equations (26), (32), (35), and (45). Other results concerning the cloud spectrum are summarized in Table 1 and shown pictorially in Figures 1, 2, and 3. The ionization rate in the WIM is $\zeta = 1.1 \times 10^{-13}$ s^{-1}. The cloud evaporation parameter is $\Sigma = 10^{1.68}$, which satisfies the requirement given in § III*d* that $\Sigma < 10^{1.75}$ for energy balance within the galactic disk.

We turn now to a calculation of the global values of $\langle n_e^2 \rangle_{\rm ISM}$ and $\langle n_e \rangle_{\rm ISM}$. The mean square electron density is

$$\langle n_e^2 \rangle_{\rm ISM} \equiv \langle n_e^2 f_w \rangle = \frac{\epsilon_{\rm UV}}{\alpha_{\rm rec}} \tag{50}$$

from equation (45); it is independent of the pressure, and for $\epsilon_{-15} = 2$ it is 0.0065 cm^{-6}. The mean electron density $\langle n_e \rangle_{\rm ISM} = \langle n_e f_w \rangle$ can be written as

$$\bar{n}_e = 10^{-1.27}\left[\frac{T_{w4}^{1.82}\epsilon_{-15}(1 + x_w)\bar{n}}{k_l \ln(a_{0u}/a_{0l})}\right]^{1/2}\left\langle\frac{1}{\tilde{P}_4^{1/2}}\right\rangle \tag{51}$$

with the aid of equations (38), (45), and (49). The average of the inverse root of P in equation (51) can be found from the theory in § III to be $\langle \tilde{P}_4^{-1/2} \rangle = 10^{0.22} Q_c^{-0.11}$. We thus find $\bar{n}_e = 0.041$ cm^{-3}. These derived numbers will be compared with observations in § V. We note here several checks of self-consistency. The various length scales defined in § III*b* are computed to be $R_c = 10^{2.26}$ pc, $H = 10^{2.30}$ pc, $R_E = 10^{2.45}$ pc, $R_{ov} = 10^{2.32}$ pc, and $\lambda_c = 10^{1.94}$ pc. Thus cooling will occur, but dense shells will not have an opportunity to form except when shocks intersect clouds for $R > R_c$. The galaxy is optically thick to UV radiation ($\lambda_w/H = 10^{-1.2}$), and losses of shock energy to the galactic halo will be significant but not dominant ($R_c \sim H > \lambda_c$). Cloud crushing is a significant ($\sim 30\%$) loss but not a dominant effect since the cloud (warm + cold) filling factor is less than 0.3; one can

TABLE 1

INTERSTELLAR CLOUD PROPERTIES FOR TYPICAL CONDITIONS

Average Cloud Properties	Cold Cores	Warm Ionized Component
Hydrogen density, n (cm^{-3})	42	0.25
Fractional ionization, x	...	0.68
Assumed temperature, T (K)	80	8,000
Filling factor, f	0.024	0.23
Intercloud distance, λ (pc)	88	12
Cloud radii (pc):		
Largest	10.0	10.8
Mean*	1.6	2.1
Smallest	0.38	2.1
Column densities (10^{19} cm^{-2}):		
Largest	173	.22
Mean*	27	.22
Smallest	0.5	.22

* Weighted by cloud area.

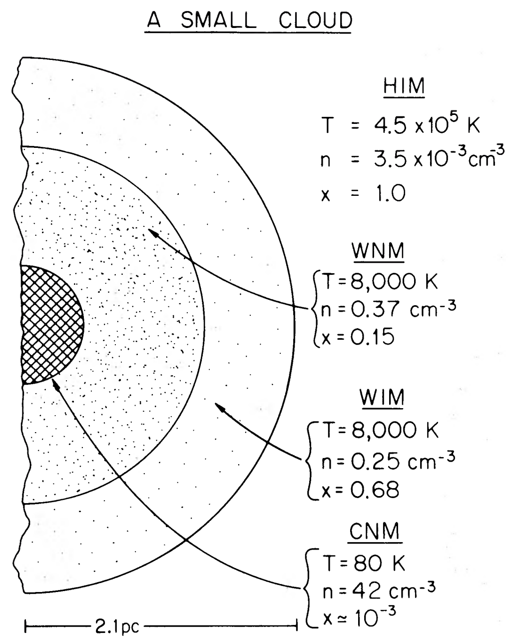

FIG. 1

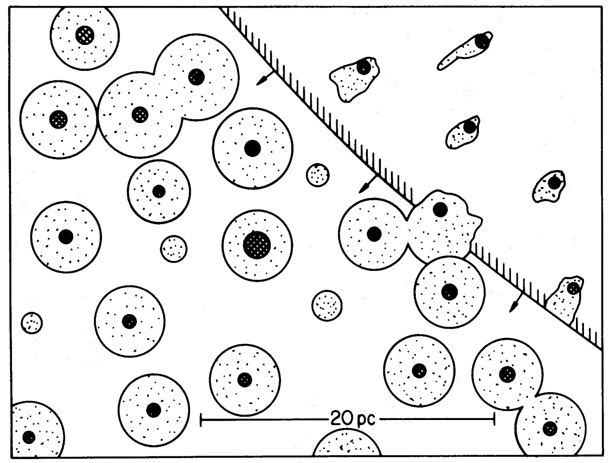

FIG. 2

FIG. 1.—Cross section of a characteristic small cloud. The crosshatched region shows the cold core, which gives the usual optical absorption lines. Next is the warm neutral medium (WNM) with ionization produced by soft X-ray background. The outer layer (WIM) is gas largely ionized by stellar UV background. Typical values of hydrogen density n, temperature T, and ionization $x = n_e/n$ are shown for each component, except that a higher than average value of the soft X-ray flux has been assumed in order to produce a significant amount of WNM at this pressure.

FIG. 2.—Small-scale structure of the interstellar medium. A cross section of a representative region 30 pc $\times$ 40 pc in extent is shown, with the area of the features being approximately proportional to their filling factors. A supernova blast wave is expanding into the region from the upper right. The radius of the neutral cores of the clouds (represented by crosshatching) ranges from about 0.4 to 1 pc in this small region; all the clouds with cores have warm envelopes (*dotted regions*) of radius $a_w \sim 2.1$ pc. A few clouds are too small to have cores. The envelopes of clouds inside the SNR are compressed and distorted.

compensate for it in the previous work in this paper by simply decreasing the assumed supernova energy E_{51} by $\sim 30\%$, a change which would have negligible effect on any of the calculated quantities.

b) *Warm Neutral Medium*

We estimate from Chevalier's (1974) calculations that soft X-ray photons in the energy range 40–120 eV ($h\bar{\nu} = 60$ eV) are produced in amount $\epsilon_x = 1.1 \times 10^{-16} S_{-13} E_{51}$ photons cm^{-3} s^{-1}. These will penetrate through the

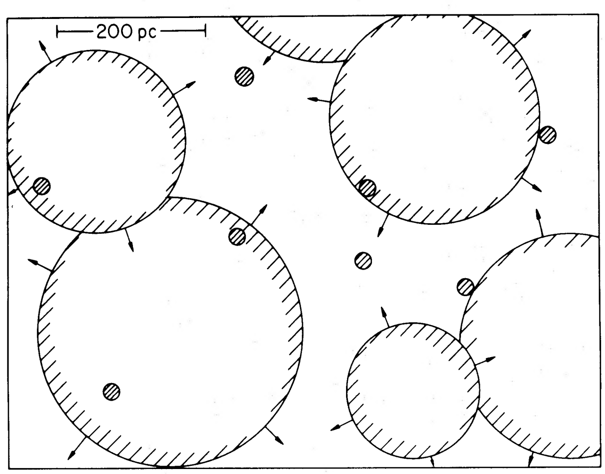

FIG. 3.—Large-scale structure of the interstellar medium. The scale here is 20 times greater than in Fig. 1: the region is 600 $\times$ 800 pc. Only SNRs with $R < R_c = 180$ pc and clouds with $a_0 > 7$ pc are shown. Altogether about 9000 clouds, most with $a_w \sim 2.1$ pc, would occur in a region this size.

warm ionized medium discussed in the last section and may, under certain circumstances, produce an intermediate layer surrounding the cold clouds which is largely neutral but maintained at a temperature of ~8,000 K. (Photoelectric heating due to grains may also contribute the formation of the WNM.) Let us assume that such layers form and that they have a filling factor of $f_{\rm WNM}$ compared with $f_{\rm WIM} = 0.23$ found in § V*a*. Further, suppose (as can be shown) that each individual layer is optically thin. At 60 eV, the photoabsorption cross section per hydrogen nucleus is $\sigma_{\rm H} = 10^{-19.06}(2.6 - x)$ cm^2, where x is the fractional ionization and neutral helium is the dominant absorber. The absorption probability for a photon traversing a distance ds is

$$d\tau = (n_{\rm WNM}\sigma_{\rm WNM}f_{\rm WNM} + n_w\sigma_w f_w + \lambda_c^{-1})ds\,, \tag{52}$$

where the three terms refer to the WNM, the WIM, and the CNM, respectively; $\sigma_{\rm WNM}$ is the value of $\sigma_{\rm H}$ in the WNM, etc. The primary photoionization rate of hydrogen ζ is the fraction of the photons absorbed by the WNM times the fraction absorbed by hydrogen times the production rate per neutral hydrogen atom:

$$\zeta = \left(\frac{n_{\rm WNM}\sigma_{\rm WNM}f_{\rm WNM}}{n_{\rm WNM}\sigma_{\rm WNM}f_{\rm WNM} + n_w\sigma_w f_w + \lambda_c^{-1}}\right)\left(\frac{1 - x_{\rm WNM}}{2.6 - x_{\rm WNM}}\right)\left(\frac{\epsilon_{\rm X}}{n_{\rm WNM}(1 - x_{\rm WNM})f_{\rm WNM}}\right). \tag{53}$$

Now $n_w f_w$ depends only weakly on the ambient pressure (see eq. [38]), and λ_c is sufficiently large that it represents an average; hence we use typical values for these quantities. Pressure equilibrium implies $n_{\rm WNM} = 1.14\tilde{P}_4(1 + x_{\rm WNM})^{-1}$. Noting that most of the dependence of ζ upon $x_{\rm WNM}$ cancels; and, anticipating that $x_{\rm WNM} \sim 0.1$, we neglect $x_{\rm WNM}$ and obtain

$$\zeta = 7.3 \times 10^{-16}(1 + 19.6\tilde{P}_4 f_{\rm WNM})^{-1}\ {\rm s}^{-1}\,.$$

For $Q > Q_c = \frac{1}{2}$, the pressure is $P = 1.67 \times 10^{-13}Q^{-0.9}$ dyn cm^{-2} so that

$$\frac{P}{\zeta} = \frac{230}{Q^{0.9}}\left(1 + 2.37\frac{f_{\rm WNM}}{Q^{0.9}}\right) \quad (Q > Q_c)\,. \tag{54}$$

The calculations of Bergeron and Souffrin (1971) indicate that a warm medium heated by 60 eV photons can exist only if $P/\zeta < (P/\zeta)_{\rm max} \approx 200$. However, heavy-element depletion will decrease the cooling rate and raise $(P/\zeta)_{\rm max}$. A reasonable value for $(P/\zeta)_{\rm max}$ including the effects of depletion is 600 (this has been confirmed in calculations by J. Bergeron, private communication 1977). Then regions with $\frac{1}{2} < Q < 1$ have a low enough pressure that a warm neutral medium exists. Assuming that $P/\zeta = (P/\zeta)_{\rm max}$ for $Q > Q_c$ and integrating over Q, we find $f_{\rm WNM} = 0.18$. This value is very approximate because the long recombination times make the assumption of local ionization equilibrium doubtful. However, one can show on energetic grounds that $f_{\rm WNM} < 0.2$.

A typical value for the ionization of the WNM can be found by using $\tilde{P} = \tilde{P}(Q = 0.75)$ and including the secondary ionizations. We obtain $x_{\rm WNM} \approx 0.15$ and, since $n_{\rm WNM} = 0.16$ cm^{-3} at this pressure, $\langle n_e\rangle_{\rm ISM} = 4 \times 10^{-3}$ cm^{-3} and $\langle n_e^2\rangle_{\rm ISM} \approx 1.0 \times 10^{-4}$. As remarked above, these values are quite approximate. The argument based simply on energetics indicates that the filling factor of the warm neutral medium will be $f_{\rm WNM} = 0.10$–0.15, reducing the contribution to $\langle n_e\rangle_{\rm ISM}$ by ~1.4 and that to $\langle n_e^2\rangle_{\rm ISM}$ by ~2.

c) *Cloud Acceleration*

Finally, we can calculate the rate at which supernova shocks will accelerate clouds, and, balancing this energy input with the losses due to inelastic cloud-cloud impacts, we can determine the equilibrium rms cloud velocity that we would obtain if only those processes occurred which are described in this paper.

The major input is due to cloud-shell collisions in the postcooling interval ($Q_c < Q < 1$), which we shall assume to be completely inelastic and calculate the energy input in the limit that $\mathscr{M}_{\rm cl} \equiv$ (mass/area)$_{\rm cloud} \gg \mathscr{M}_{\rm sh} \equiv$ (mass/area)$_{\rm shell}$, and $v_{\rm shell} \gg v_{\rm cl}$. Then the amount of kinetic energy deposition per SNR can be shown to be

$$\Delta E_{\rm in,late} = 2\pi R^2 dR M_{\rm cl}(a_0)N(a_0)da_0\,\frac{\mathscr{M}_{\rm sh}}{\mathscr{M}_{\rm cl}}\left(\frac{\mathscr{M}_{\rm sh}}{\mathscr{M}_{\rm cl}}\,v_{\rm sh}^2 - \frac{1}{3}\,v_{\rm cl}^2\right) \tag{55}$$

after averaging over the angle between ($\boldsymbol{v}_{\rm sh}$, $\boldsymbol{v}_{\rm cl}$) for clouds of mass in $a_0 \to a_0 + da_0$ and a supernova (with $\mathscr{M}_{\rm sh}$, $v_{\rm sh}$) expanding in radius $R \to R + dR$. Noting that the first term dominates over the "Fermi" term (negative here because of assumed inelasticity), and averaging over the assumed cloud spectrum gives

$$\langle\Delta E\rangle_{\rm in,late} = \frac{3\pi^2}{2}\frac{N_{\rm cl}}{\rho_w}\,a_{wb}R^2 dR\mathscr{M}_{\rm sh}^2 v_{\rm sh}^2\,. \tag{56}$$

We have assumed magnetic forces are sufficiently weak that the warm envelopes are compressed and stripped from the cold cores, thereby producing new small clouds. We must now integrate this between R_c and R_{ov}. Since, by coincidence, it occurs that $\Sigma(R_c) \approx \lambda_{\rm WIM}(n_h)_{\rm typ}$, the shell density will be nearly constant during this phase at the postcooling value. Thus

$$\int_{R_c}^{R_{ov}} R^2 \mathscr{M}_{\rm sh}{}^2 v_{\rm sh}{}^2 dR = [\mathscr{M}'(R_c)]^2 R_c{}^3 v_c{}'^2 \int_1^{R_{ov}/R_c} x^{2(2\eta'-1)/\eta'} dx\,.$$

We take the integral to be $\frac{1}{8}$ for $\eta' = 0.39$ and $Q_c = 0.5$, giving

$$\langle\langle \Delta E\rangle\rangle_{\rm in,late} = \frac{3\pi^2}{16}\frac{N_{\rm cl}}{\rho_w} a_{wb}[\mathscr{M}'(R_c)]^2 R_c{}^3 v_c{}^3 = 5.2 \times 10^{49} \text{ ergs per SN}\,, \tag{57}$$

or about 5% of the available energy.

There is also acceleration of clouds in early phases ($0 < Q < Q_c$) (Cowie, McKee, and Ostriker 1977); neglecting this contribution, we find

$$\frac{d\langle\langle \dot{E}\rangle\rangle_{\rm in}}{dV} = 6.2 \times 10^{-27} \text{ ergs cm}^{-3}\text{ s}^{-1} \quad (S_{-13} = 1, E_{51} = 1)\,. \tag{58}$$

The energy lost in an inelastic collision between two clouds (1, 2), $\Delta \boldsymbol{v} = \boldsymbol{v}_1 - \boldsymbol{v}_2$, is

$$\Delta E = -0.5|\Delta \boldsymbol{v}|^2 m_1 m_2/(m_1 + m_2)\,.$$

Taking the collision cross section to be one-half of geometrical, $\frac{1}{2}\pi(a_1 + a_2)^2$, and averaging over a Gaussian distribution of cloud velocities, we find

$$\frac{d\langle\langle \dot{E}\rangle\rangle_{\rm out}}{dV} = -2.72 N_{\rm cl}{}^2 v_{\rm rms}{}^3 \left\langle \left[\frac{m_1 m_2}{m_1 + m_2}\right](a_1 + a_2)^2 \right\rangle. \tag{59}$$

For the assumed cloud distribution the averaged quantity is approximately $(8\pi/3)\rho_c a_{0l}{}^3 a_l{}^2$, giving

$$\frac{d\langle\langle \dot{E}\rangle\rangle_{\rm out}}{dV} = -22.8 \rho_c a_{0l}{}^3 a_l{}^2 N_{\rm cl}{}^2 v_{\rm rms}{}^3 = -1.26 \times 10^{-29} v_{\rm rms,5}{}^3 \text{ ergs cm}^{-3}\text{ s}^{-1}\,. \tag{60}$$

Because of the possible separation of cores from clouds during collisions this may somewhat overestimate $E_{\rm out}$. Then balancing input and output gives

$$\frac{d\langle\langle \dot{E}\rangle\rangle_{\rm out}}{dV} + \frac{d\langle\langle \dot{E}\rangle\rangle_{\rm in}}{dV} = 0\,, \qquad v_{\rm rms} = 7.9 \text{ km s}^{-1}\,, \tag{61}$$

which is close to the value 10 km s^{-1} given in Spitzer (1977).

While it has been known for some time that supernovae are energetically capable of keeping the clouds stirred, the present treatment provides a simple and specific physical mechanism for effectuating the energy transfer; we will return to a more careful treatment of this problem in a subsequent publication (Cowie, McKee, and Ostriker 1977).

d) *Uncertainties*

Since all of the quantities derived in this section are in fairly good agreement with observations, we owe the reader a word about uncertainties in the model and how they would have affected the computation.

First the cloud spectrum: Could we not have simply taken a single standard two- or three-layer cloud rather than the more complicated distribution $N(a_0) \propto a_0{}^{-4}$? And, if so, would the results have been materially different? The calculation was performed and the results are plausible, the cold cores being similar to (but smaller than) Spitzer's "standard clouds" and the filling factors the same (as they must be) since they are determined by the geometrical model and the values of $\bar{n}$, ϵ, and $\tilde{P}$ adopted. As small objections to such a simple model, we note that the deficiency of small clouds would make it impossible to satisfy the evaporation condition and the larger values of λ_w found might make the computed degree of inhomogeneity greater than is observed. However, the primary virtue in adopting a spectrum was that it is intrinsically plausible to have a range in cloud sizes and that the theory then automatically restricts the possible sizes to a moderately narrow band surrounding the typical "standard clouds." Of the various other parameters, ($\bar{n}$, T_w, T_c) are fairly well known, P is computed self-consistently, but $\epsilon_{\rm UV}$, $\epsilon_{\rm X}$, $(P/\zeta)_{\rm max}$ are poorly known and $f_{\rm WIM}$ depends linearly on the first of these and $f_{\rm WNM}$ depends quadratically on both the last two and so should be considered quite insecure. The details of the cloud acceleration modeled here may not be accurate, but it is difficult to avoid having supernova shocks inject several percent of their energy into cloud

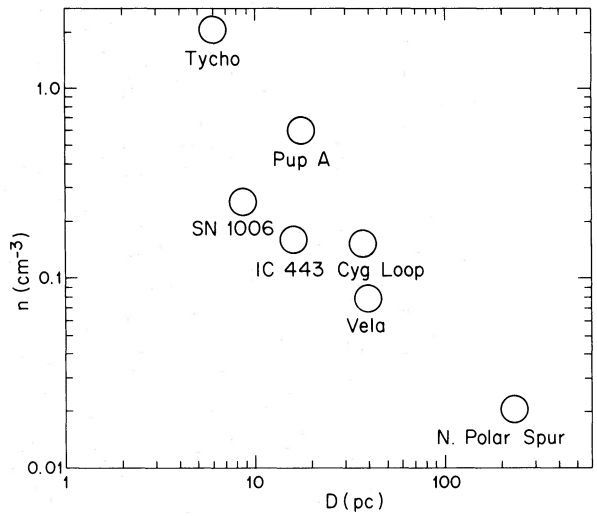

FIG. 4.—Mean (rms) density within galactic supernova remnants derived from X-ray flux, temperature, and estimated distance plotted versus diameter of remnant from angular size and distance. Data from Gorenstein and Tucker (1976) and Hayakawa *et al.* (1976). Observed trend consistent with evaporation theory, although observational selection may be important.

motions given the adopted geometrical model. This input, as Spitzer (1968) notes, will necessarily make a substantial contribution to the maintenance of cloud motions.

VI. COMPARISON WITH OBSERVATIONS

a) *SNR Properties*

Much of this paper depends heavily on the theory of evolution of supernova remnants dominated by evaporation of clouds presented in § II. There we predicted that, for supernovae of given energy, exploding in a given medium, the density will fall as $R^{-5/3}$ and the temperature as $R^{-4/3}$. This contrasts strikingly with the behavior in the Sedov solution of $n \propto R^0$, $T \propto R^{-3}$. Gorenstein, Harnden, and Tucker (1974) commented on the fact that the slope observed in the temperature versus radius relation was much flatter than anticipated from the Sedov solution and suggested various possible interpretations. Here we show in Figure 4 the density versus radius relation using data from Gorenstein and Tucker (1976) excluding Cas A, supplemented by a recent observation of the North Polar Spur (Hayakawa *et al.* 1977). The trend of lower densities observed for larger SNR is unmistakable. It can possibly be accounted for partially by selection, and quibbles with individual data points are also possible, but the data clearly seem consistent with a model in which the density is large inside supernova remnants in early phases (near $\bar{n} \approx 10^0$ cm^{-3}) due to effective evaporation of clouds and in which it drops with increasing age and size towards the ambient value ($n_0 \approx 10^{-2.5}$ cm^{-3}) as evaporation lessens in importance.

b) O VI *Absorption Lines*

Both the individual O VI line widths and the dispersion of the mean O VI velocities along different lines of sight are of order 20 km s^{-1} (Jenkins and Meloy 1974). This is much less than the typical isothermal sound speed in the HIM (80 km s^{-1}) and makes it unlikely that the *observed* lines arise in shock fronts or in the general HIM, which is probably quite turbulent. Instead, we suggest that the observed lines arise in the conductive interfaces around clouds (Castor, McCray, and Weaver 1975 have proposed a circumstellar version of this idea). The velocity dispersion of individual O VI components should be somewhat greater than the cloud velocity dispersion because of the outflow velocity of the evaporating gas, as is indeed observed (Jenkins 1977).

The column density of O VI in a conductive interface in which classical evaporation is occurring is $\mathcal{N}(\text{O VI}) = \int n(\text{O VI})dr = 7.0 \times 10^{13} T_{f6}^{-3/2} n_f a_{\text{pc}}$ (McKee and Cowie 1977), where T_f and n_f are the temperature and density far from the cloud. In a cloudy medium T_f is related to the mean temperature T_h by $T_f \approx (1 - f_{\text{cl}}^{1/3})^{-3/5} T_h$, and n_f is determined by pressure equilibrium ($n_f T_f = n_h T_h$). The mean density of O VI due to the conductive interfaces is then equal to the number of clouds per unit volume N_{cl} times the number of O VI ions per cloud: $n(\text{O VI}) = N_{\text{cl}}(4\pi/3)(a_2^2 + a_2 a_1 + a_1^2)\mathcal{N}(\text{O VI})$, where the O VI has a significant concentration from a_1 to a_2. Using the results of § V, we find that $\mathcal{N}(\text{O VI}) = 4.1 \times 10^{11}$ cm^{-2}, so that the O VI column density through both sides of the cloud is typically $\sim 10^{12}$ cm^{-2}. Since the concentration of O VI peaks at $10^{5.5}$ K, we estimate a_1, a_2 as the radii at which $T = 10^{5.5} \times 2.718^{\mp 1/2}$; we find $a_1 = 1.03a$ and $a_2 = 1.43a$, so that $n(\text{O VI}) = 6.6 \times 10^{-8}$ cm^{-3}, slightly more than twice the mean value observed in the plane of the Galaxy, 2.8×10^{-8} cm^{-3} (Jenkins 1977). Since our result varies as the cube of the somewhat uncertain mean cloud radius $\langle a \rangle = a_{wb}$, this degree of agreement is satisfactory.

In addition, since much of the Galaxy is filled with hot gas, we expect a relatively large value of $\langle n(\mathrm{O\ VI})\rangle$ from the ambient medium. A straightforward application of the previous theory gives $\langle n(\mathrm{O\ VI})\rangle = 9 \times 10^{-8}$ cm^{-3} from the HIM, with 40% coming from regions having $R < R_c$ and 60% from those having $R_c < R < R_{ov}$. The line widths will be 100–200 km s^{-1}, so this emission may be difficult to detect without special efforts.

c) Soft X-Rays

The temperature and density we calculate for the HIM are in excellent agreement with the values inferred from observation by Levine *et al.* (1977). The average emissivity of space due to SNRs with $R < R_c$ is $(35/16) j_c Q_c$, where j_c is the emissivity at $R = R_c$ and where we have taken $\Lambda \propto T^{-0.6}$. The space outside such remnants contributes an additional 10% to the emissivity so that altogether $\bar{j} = 2.41 j_c Q_c$. Define $T_{h,\mathrm{med}}$ such that half the emission occurs at $T_h > T_{h,\mathrm{med}}$ and half at $T_h < T_{h,\mathrm{med}}$; and let $n_{eh,\mathrm{rms}} = (\int n_{eh}^2 dQ)^{1/2}$ be the rms electron density in the HIM. We find

$$T_{h,\mathrm{med}} = 10^{5.76} Q_c^{-0.15} = 6.3 \times 10^5\ \mathrm{K}\,,$$

$$n_{eh,\mathrm{rms}} = 10^{-2.15} Q_c^{-0.04} = 7.3 \times 10^{-3}\ \mathrm{cm}^{-3}\,, \tag{62}$$

which are almost identical to the values favored by Levine *et al.* (1977), $T = 6 \times 10^5$ K and $n_e = 0.007$ cm^{-3}. Since less than ~40% of the space contributes more than ~90% of the emission, we expect—as is observed—a patchy distribution for the soft X-ray emission.

The situation is in fact a good deal more complicated because the results of Levine *et al.* depend fairly sensitively upon interstellar absorption and because a number of observations (Burstein *et al.* 1976; Cash, Malina, and Stern 1976, and de Korte *et al.* 1976) suggest that the spectrum of the diffuse soft X-ray emission is too complex to be accounted for by a single temperature.

We begin by looking at the question of interstellar absorption. Bowyer and Field (1969) have analyzed the opacity of a cloudy medium. Their results can be put into a more useful form by approximating $[1 - \exp(-\tau_c)]/\tau_c \approx (1 + \tau_c)^{-1}$, which is accurate to within 30%; this gives

$$\frac{k_{\mathrm{x}}}{\bar{n}_c \sigma} = \frac{1}{1 + \langle a_c^2 \tau_c \rangle} = \frac{1}{1 + \bar{n}_c \sigma \lambda_c}\,, \tag{63}$$

where k_{x} is the effective opacity of the cloudy gas and σ the X-ray absorption cross section. In other words, the mean free path for X-ray absorption is the sum of that in a uniform medium $(\bar{n}_c \sigma)^{-1}$ and the cloud mean free path λ_c. The area weighted cloud opacity $\langle a_c^2 \tau_c \rangle$ is about $(0.28/E_k)^3$, where E_k is the photon energy in keV.

Making a similar analysis for the WIM, we find that the total opacity of the ISM is

$$k_{\mathrm{x}} = \left(\lambda_c + \frac{1}{\bar{n}_c \sigma}\right)^{-1} + \left(\lambda_w + \frac{1}{\bar{n}_w \sigma}\right)^{-1}, \tag{64}$$

where the WIM term exceeds the cloud term below 0.11 keV.

We shall compare our results with the data of Burstein *et al.* (1976), since their data is in a form which is most suitable for analysis. Their energy bands correspond to typical photon energies of about 170 eV (B band), 230 eV (C band), and 400–850 eV (M band). Optical depth unity is reached at a column density of only 5×10^{19} cm^{-2} for photons in the B band, and they suggest that the large observed flux at that energy implies we are embedded in a "tunnel of hot gas" (i.e., a SNR). The X-ray intensity in a direction in which the distance and total optical depth to the edge of the SNR are d and $\tau = k_c d$ is

$$I = \frac{\langle n_e^2 \rangle \Lambda d}{4\pi\tau}(1 - e^{-\tau})\,. \tag{65}$$

For the two low-energy bands, τ is determined primarily by the cloud spacing λ_c and hence is relatively insensitive to energy. Knowing $\langle n_e^2 \rangle d$ and τ, one can use the results of Burstein *et al.*, to infer the number of counts per second they would have observed in each of their energy bands.

As an example, assume we are located in a SNR with $T = 10^{5.8}$ K. Then $n = 10^{-2.14}$ cm^{-3}, $\tilde{P} = 10^{4.03}$ K cm^{-3}, $R = 10^{2.14}$ pc, and $Q = 0.14$. The optical depth in each band is 1.27 (B), 1 (C), and 0.28 (M), and the predicted number of counts per second in each band is 60 (B), 100 (C), and ~2 (M), which is to be compared with the values typically observed in the plane 40 (B), 120 (C), and 100 (M). The local SNR model thus accounts for the low-energy data but appears to fail for the high-energy data. In fact, both Burstein *et al.* (1976) and de Korte *et al.* (1976) have suggested that an additional, hotter component ($T \sim 10^{6.3}$ K) is needed to account for the 0.4–0.85 keV data. SNRs with $T \sim 10^{6.3}$ K have a very small filling factor ($Q \sim 2.5 \times 10^{-3}$) and are unlikely to be able to account for the data. However, these arguments all depend on the use of steady state calculations of the X-ray emissivity, which may underestimate the rate by as much as an order of magnitude (cf. Shapiro and Moore 1976). Such time-dependent effects should be particularly important in SNRs because both adiabatic expansion and conductive cooling accelerate the cooling over the isochoric radiative value used by Shapiro and Moore. It is thus possible that our results can account for the M-band data of Burstein *et al.* as well, although detailed calculations are necessary to verify this. Additionally, coronal gas from the galactic gaseous halo (cf. § III*e*) may make a significant contribution to the higher energy observations.

d) *Interstellar Clouds*

It is of some significance that the model presented here attempts to derive, from first principles, the pressure of the interstellar medium. We found the typical pressure to be 3700 K cm^{-3} (eq. [49]), almost identical to the mean value found by Jura (1975) from an analysis of molecular hydrogen in six clouds not near H II regions, and comparable to the value 2000 K cm^{-3} usually estimated from 21 cm observations (Field 1975). The constraint imposed by Shapiro and Field (1976) is avoided because the X-ray emission arises in the high-temperature, high-pressure regions inside SNRs prior to cooling, whereas the observed O VI arises in cooler regions.

The values of $\langle n_e \rangle_{\mathrm{ISM}}$ and $\langle n_e^2 \rangle_{\mathrm{ISM}}$ (0.041, 0.0065) in § V*a* are near the values found observationally from pulsar dispersion measures and diffuse Hα emission measures (Spitzer 1977), although the first number is perhaps somewhat too large.

As noted previously, the cloud velocity dispersion produced by passage of supernova shocks, 8 km s^{-1}, agrees with observations, as does the number of cold clouds per kiloparsec with Hobbs's (1974) high-resolution studies. Although we adopted a distribution of the same form as Hobbs, the agreement of λ_c with his observations is not trivial since N_{cl} was determined largely by the evaporation theory and the required value of Σ.

The geometric model adopted has certain consequences for absorption line studies: whenever there is a cold cloud along a given line of sight, at a given velocity, there should be warm ionized material as well at the same velocity and often ($\sim\frac{1}{3}$ of cases) warm, largely neutral material as well. The latter should show relatively high states of ionization since only photons with $E > 13.6$ eV can reach it. This correlation of properties definitely would not be expected if the warm medium were intercloud gas as in the picture of Field, Goldsmith, and Habing (1969). Dickey, Salpeter, and Terzian's (1977) 21 cm observations are consistent with such a correlation.

Probably the greatest weakness of the model presented here concerns the warm neutral medium. Radhakrishnan *et al.* (1972) and others have interpreted 21 cm observations to indicate that a major fraction of the hydrogen at high latitudes has a spin temperature in excess of 750 K. For the ISM as a whole, Spitzer (1977) gives 14% as the fraction of the gas which is warm and neutral. We find only 4% of the medium warm and neutral in the plane, with one-half in the WIM and one-half in the WNM. This fraction will increase somewhat with the inclusion of photoelectric heating, and it will rise substantially with altitude above the plane, but it remains to be demonstrated that our model gives a value consistent with observation.

Finally we note that the column density to nearby stars (<100 pc) of neutral gas is expected to be quite low. Since it is unlikely that the line of sight will pass through any cold clouds, we would expect from Table 1 about eight warm components per 100 pc or 1.8×10^{19} H atoms. The neutral fraction $(1 - x_w)$ is 0.3, giving 0.017 neutral H atoms cm^{-3} on average for lines of sight not passing through cold clouds, in agreement with the low values found by Bohlin (1975) and Margon *et al.* (1976) for absorption between us and nearby hot stars.

One theoretical point might be noted here. The very existence of cold but not self-gravitating clouds is somewhat puzzling. As pointed out by Spitzer (1968, p. 182), the clouds would have extremely short lifetimes if not maintained in pressure equilibrium with an intercloud medium. But the maximum density of an *ionized* intercloud medium is given by the pulsar dispersion measure $n_{hc} \leq 4 \times 10^{-2}$. If this intercloud medium had a temperature of $\sim 10^4$ K (rather than $\sim 10^6$ K), pressure balance would fail by two orders of magnitude and the cold clouds would rapidly disperse. The requirement of pressure equilibrium would have allowed one to predict that the intercloud medium must be quite hot (or largely neutral [Field, Goldsmith, and Habing 1969]) even in the absence of a definite mechanism (supernovae) to maintain that temperature or direct observations of the hot gas (soft X-rays and O VI).

This work has been in progress for several years and has benefited greatly from the exposure that the authors have had to the wisdom of colleagues expert in the study of the interstellar medium at the Center for Astrophysics, Princeton University Observatory, and the University of California. In particular, they must single out Drs. Lennox Cowie and Lyman Spitzer, Jr., for careful readings made of various drafts of the manuscript and numerous helpful suggestions. Dr. Jacqueline Bergeron kindly supplied a calculation of an X-ray heated cloud with depleted abundances, and Dr. Edward Jenkins made a number of helpful comments on the interpretation of the O VI and soft X-ray observations. Material support was provided by National Science Foundation grants MPS 74-18970 and AST 76-20255 and AST 75-02181.

APPENDIX A

GLOSSARY OF UNFAMILIAR SYMBOLS USED REPEATEDLY

a_0: equivalent radius of cloud in absence of ionization $= M_{\mathrm{cl}}/(4\pi/3)\rho_c$, where ρ_c is density of cold cloud material; unit = pc.
a_c: radius of neutral core if any; unit = pc.
a_w: radius of warm, ionized envelope; unit = pc.

a_{wb}, a_{ob}: radius and equivalent radius at which neutral core vanishes (cf. eq. [C1]), unit = pc.
a_u: upper limit on cloud radius, unit = pc.
a_{wl}, a_{0l}: lower limits on cloud radii, unit = pc.
c_h: isothermal sound speed of SNR interior.
CNM: cold, neutral interstellar medium.
E, E_{51}: supernova energy, same in unit of 10^{51} ergs.
$E_{\rm th}$: supernova thermal energy.
f_w, $f_{\rm SNR}$, etc.: filling factors of warm medium, SNR, etc.
H: galactic cloud scale height, unit = pc.
HIM: hot ($\sim 10^{5.5}$ K) interstellar medium.
J: mean ionizing photon density, unit = photons $\rm cm^{-2}\, s^{-1}\, sr^{-1}$.
k_l: dimensionless constant in cloud spectrum discussion (eq. [30]).
M: mass of SNR.
$\dot{M}_{\rm ev}$, $\Delta M_{\rm ev}$: evaporation rate of cloud, unit = g $\rm s^{-1}$; evaporated mass, unit = $M_\odot$.
M_s: mass of shell, post-cooling.
$\mathscr{M}_{\rm sh}$, $\mathscr{M}_{\rm cl}$: mass of SNR shell, cloud per unit area, unit = y $\rm cm^{-2}$.
$N_{\rm cl}$: number density of clouds; unit = $\rm pc^{-3}$.
n_0: H atom density of ambient interstellar medium, unit = $\rm cm^{-3}$.
n_h: H atom density of SNR interior, unit = $\rm cm^{-3}$.
n_{hc}: H atom density of SNR interior at cooling.
n_c, n_w: H atom density in cold cloud cores, warm ionized envelopes.
n_e: electron density.
$\mathscr{N}$: atoms $\rm cm^{-2}$ in column.
P: pressure.
$\tilde{P}$: pressure, $\tilde{P} \equiv P/k$.
P_0: ambient interstellar pressure.
P_{hc}, P_{hc}': pressure in SNR just before or after cooling.
Q, Q_c, $Q_{\rm SNR}$: $dQ(R)$ is probability that an arbitrary point is in SNR with R between R and $R + dR$ (eq. [1]); $Q_c = Q(R_c)$, $Q_{\rm SNR} = Q(R_E)$.
R: SNR radius, unit = pc.
R_c: SNR radius at cooling, unit = pc.
R_E: SNR radius when equilibrium reached with ambient pressure, unit = pc.
R_{ov}: SNR radius when collision occurs with similar SNR, unit = pc.
S, S_{-13}: rate of SN explosions, rate in units of 10^{-13} $\rm pc^{-3}\, yr^{-1}$.
SNR: supernova remnant.
T_h: temperature of SNR interior, unit = K.
T_{hc}: temperature of SNR interior at cooling point, unit = K.
T_c, T_w: temperatures of cold cloud cores and warm envelopes.
t_c: age of SNR at dense shell formation, unit = yr.
typ: typical conditions in HIM; cf. eqs. (25) and (26).
V: volume of SNR, unit = $\rm pc^3$.
v_b, v_{bc}: velocity of SNR blast wave, same at cooling.
$v_{\rm cl}$, $v_{\rm rms}$: velocity of cloud, rms cloud velocity in ISM.
WIM: warm ($\sim 10^4$ K) ionized ISM.
WNM: warm ($\sim 10^4$ K) neutral ISM.
x: parameter measuring importance of evaporated mass, eq. (6).
x_w: fractional ionization in warm medium.
α: ratio of SNR blast wave velocity to interior isothermal sound speed.
$\alpha_{\rm rec}$: H recombination rate, unit = $\rm cm^3\, s^{-1}$.
β: ratio of cooling rate in SNR to that in equivalent uniform-density sphere.
δ: fraction of mass deposited in shell at cooling.
$\epsilon_{\rm UV}$, $\epsilon_{\rm X}$: mean production rate of UV or soft X-ray photons, unit = $\rm cm^{-3}\, s^{-1}$.
ζ: primary ionization rate per H atom, unit = $\rm s^{-1}$.
η, η': parameter in evaporation theory, cf. eq. (6); $R \propto t^\eta$ before cooling and $R \propto t^{\eta'}$ after cooling.
Λ: cooling function, unit = erg $\rm cm^{+3}\, s^{-1}$.
λ_c, λ_w: mean distance between cold or warm clouds along a line of sight, unit = pc.
μ, $\mu_{\rm H}$: mass of average particle, mass per H atom.
ρ_0: ambient interstellar mass density.
$\sigma_{\rm H}$: ionization cross section for hydrogen.
Σ: cloud evaporation parameter, eq. (7).
ϕ: efficiency of cloud evaporation, eq. (7).

APPENDIX B

SCALING WITH Q

In calculating mean quantities in SNRs and in the HIM it is useful to have the variables as functions of the parameter Q (eq. [1]). We assume $R \propto t^{\eta}$; then for η = const, $Q = Q_c(R/R_c)^{(3\eta+1)/\eta}$ and $v_b = \eta R/t$. For purposes of calculation, we ignore the slight variation in η given by equation (6) and set $\eta = 3/5$ prior to cooling. Then

$$\begin{aligned} R &= R_c(Q/Q_c)^{3/14}\,, \\ t &= t_c(R/R_c)^{5/3} = t_c(Q/Q_c)^{5/14}\,, \\ v_b &= v_{bc}(R/R_c)^{-2/3} = v_{bc}(Q/Q_c)^{-1/7}\,, \\ n_h &= n_{hc}(R/R_c)^{-5/3} = n_{hc}(Q/Q_c)^{-5/14}\,, \\ \tilde{P}_h &= \tilde{P}_{hc}(R/R_c)^{-3} = \tilde{P}_{hc}(Q/Q_c)^{-9/14}\,, \\ T_h &= T_{hc}(R/R_c)^{-4/3} = T_{hc}(Q/Q_c)^{-4/14}\,, \qquad (Q \le Q_c) \end{aligned} \tag{B1}$$

where R_c, etc., are given in equation (9). For example, explicit expressions for the mean density and temperature inside an adiabatic SNR are $n_h = 10^{1.43}R^{-5/3}$ and $T_h = 10^{8.65}R^{-4/3}$ K, where we have assumed typical values of the parameters (in particular, $E_{51} = 1$ and $\Sigma = 10^{1.68}$ so that $Q_c = \frac{1}{2}$). If either the SNR or the ambient ISM is atypical, then more general expressions are needed; for n_h, see equation (20), and for T_h, equation (3) gives

$$T_h = 10^{8.09}E_{51}{}^{1/3}\Sigma^{1/3}R^{-4/3}\ \mathrm{K}\,. \tag{B2}$$

These results are valid provided that (1) the SNR is evaporation-dominated (i.e., $n_h \gg n_0$), and (2) the evaporation is classical (Cowie and McKee 1977), which requires $R \gtrsim 10^{1.5}$ pc.

After cooling, a dense shell forms and the dynamics are uncertain. We denote quantities just after cooling by a prime. Assuming $\eta' = 0.39$ and that half the mass in the SNR goes into the shell, we find

$$\begin{aligned} R &= R_c(Q/Q_c)^{0.180}\,, \\ t &= t_c(R/R_c)^{2.56} = t_c(Q/Q_c)^{0.46}\,, \\ v_b &= v_{bc}'(R/R_c)^{-1.56} = 0.65v_{bc}(Q/Q_c)^{-0.28}\,, \\ n_h &= n_{hc}'(R/R_c)^{-3} = \tfrac{1}{2}n_{hc}(Q/Q_c)^{-0.54}\,, \\ \tilde{P}_h &= \tilde{P}_{hc}'(R/R_c)^{-5} = \tfrac{1}{2}\tilde{P}_{hc}(Q/Q_c)^{-0.90}\,, \\ T_h &= T_{hc}(R/R_c)^{-2} = T_{hc}(Q/Q_c)^{-0.36}\,. \qquad (Q > Q_c) \end{aligned} \tag{B3}$$

In particular $R_{ov} = R(Q = 1) = 1.13R_c = 206$ pc for $Q_c = \frac{1}{2}$.

APPENDIX C

CLOUDS WITH IONIZED EDGES

Assume the clouds are spherical. Let a_0, a_c, a_w, a_{0b}, a_{wb}, a_u, a_{0l}, a_{wl} be defined as in the Glossary. We assume that the transition from ionized to neutral gas is sharp so that a_c is well defined, and that $n_c \gg n_w$. The volume of the warm, largely ionized region is

$$\frac{4\pi}{3}(a_w{}^3 - a_c{}^3) = \frac{4\pi}{3}a_w{}^2 a_{wb} \tag{C1}$$

provided $a_c > 0$. The radius a_0, which is proportional to (cloud mass)$^{1/3}$, is given by

$$n_w(a_w{}^3 - a_c{}^3) + n_c a_c{}^3 = n_c a_0{}^3\,. \tag{C2}$$

For clouds with neutral cores ($a_c > 0$) this gives

$$a_0{}^3 = a_w{}^3 - \left(1 - \frac{n_w}{n_c}\right)a_w{}^2 a_{wb}\,; \tag{C3}$$

whereas for fully ionized clouds ($a_c = 0$) we find

$$a_0{}^3 = \left(\frac{n_w}{n_c}\right)a_w{}^3\,. \tag{C4}$$

Note that this applies at a_{0b} also, so that $a_{0b}{}^3 = (n_w/n_c)a_{wb}{}^3$. One can readily show that half the mass is in the core at $a_c = a_{0b}$, $a_w = (1 + n_w/n_c)a_{wb}$.

The characteristics of the clouds which must be evaluated are the filling factors

$$f_c = \int N(a_0)\frac{4\pi}{3}a_c{}^3 da_0 \propto \langle a_c{}^3\rangle \tag{C5}$$

and

$$f_w = \int N(a_0)\frac{4\pi}{3}a_w{}^3 da_0 - f_c\,; \tag{C6}$$

the area-weighted column density for the neutral cores

$$\mathscr{N}_c = \frac{4}{3}n_c\frac{\langle a_c{}^3\rangle}{\langle a_c{}^2\rangle}\,; \tag{C7}$$

the mean cloud separation λ_j along a line of sight both for the neutral cores (λ_c) and the entire cloud (λ_w)

$$\lambda_j{}^{-1} = \int N(a_0)\pi a_j{}^2 da_0 \propto \langle a_j{}^2\rangle\,; \tag{C8}$$

and the cloud evaporation parameter

$$\Sigma^{-1} = \frac{4\pi\phi}{\alpha}\int a_w N(a_0)da_0 = \frac{3(f_c + f_w)\phi\langle a_w\rangle}{\alpha\langle a_w{}^3\rangle}\,, \tag{C9}$$

where $N(a_0) = N_0 a_0{}^{-4}$ is the cloud distribution function and $\langle a\rangle$, etc., are moments of the distribution.

To evaluate the moments $\langle a_c{}^m\rangle$, introduce the variable y by

$$a_w = a_{wb}(1 + yn_w/n_c) \tag{C10}$$

so that $a_c{}^3 = a_{0b}{}^3 y(1 + yn_w/n_c)^2$ and $a_0{}^3 = a_{0b}{}^3(1 + y)(1 + yn_w/n_c)^2$. Neglecting $n_w/n_c \sim 5 \times 10^{-3}$ compared to unity, we find

$$\langle a_c{}^m\rangle = \frac{a_{0c}{}^3}{a_{0b}{}^{3-m}}\int_{y_l}^{y_u}\frac{y^{m/3}(1 + 3yn_w/n_c)}{(1 + y)^2(1 + yn_w/n_c)^{3-(2m/3)}}\,dy\,. \tag{C11}$$

Here the average extends only over those clouds with cores, so we have defined a_{0c} as the larger of a_{0l} and a_{0b}. For $m = 3$, this expression becomes

$$\langle a_c{}^3\rangle = a_{0c}{}^3\left\{\ln\left[\left(\frac{a_{0u}}{a_{0b}}\right)^3\left(1 - \frac{a_{wb}}{a_{0u}}\right)\right] - 1\right\} \approx 3a_{0c}{}^3\ln(a_{0u}/a_{0b})\,. \tag{C12}$$

For $m = 1, 2$ the terms proportional to n_w/n_c may be neglected and

$$\langle a_c{}^m\rangle = \frac{a_{0c}{}^3}{a_{0b}{}^{3-m}}\left[\frac{y_l{}^{m/3}}{1 + y_l} + m\int_{y_l{}^{1/3}}^{\infty}\frac{x^{m-1}dx}{1 + x^3}\right]\,. \tag{C13}$$

If the smallest clouds have no cores ($k_l \equiv a_{0l}{}^3/a_{0b}{}^3 < 1$), then $a_{0c} = a_{0b}$ and $y_l = 0$, which gives $\langle a_c\rangle = 1.21a_{0b}$ and $\langle a_c{}^2\rangle = 2.42a_{0b}{}^2$. For $k_l = 2$, as adopted in the text, one has $a_{0c} = a_{0l}$ and $y_l = 1$, which gives $\langle a_c\rangle = 1.75a_{0b}$ and $\langle a_c{}^2\rangle = 4.34a_{0b}{}^2$.

The behavior of the average outer radius of the clouds $\langle a_w\rangle$ is quite different from $\langle a_c\rangle$ because over a wide range of cloud masses ($a_{0b} < a_0 \lesssim \frac{1}{2}a_{wb}$) the outer radius is approximately fixed at a_{wb}. If $a_{0l} > a_{0b}$ so that the smallest clouds have cores, then

$$\langle a_w{}^m\rangle = a_{wb}{}^m \quad (m = 1, 2, 3)\,. \tag{C14}$$

On the other hand, if $a_{0l} < a_{0b}$ ($k_l < 1$) so that there are some fully ionized clouds, then

$$\langle a_w^m \rangle = a_{wb}^m \left(\frac{3k_l^{m/3} - mk_l}{3 - m} \right) \quad (m = 1, 2) ,$$
$$\langle a_w^3 \rangle = a_{wb}^3 k_l (1 - \ln k_l) . \tag{C15}$$

APPENDIX D

ADIABATIC, EVAPORATION-DOMINATED SUPERNOVA REMNANTS

The assumptions underlying our treatment of the adiabatic expansion of a SNR with mass injection by evaporation of clouds are discussed in the text. Equation (5a) for $R(n_h, t)$ follows from combining two expressions for the blast wave velocity: $v_b = \alpha c_h = \alpha(0.71E/2\pi\rho_h R^3)^{1/2}$, where $\alpha =$ const. and $v_b = \eta R/t$ (since $R \propto t^\eta$, where η is assumed to be slowly varying).

To complete the solution, we must find the variation of the internal density n_h with R. Mass conservation (eq. [4]) can be expressed as

$$\frac{dM}{dR} = 4\pi R^2 \rho_0 + \frac{N_{\rm cl}\dot{M}_{\rm ev}}{\alpha c_h}\left(\frac{4\pi R^3}{3}\right) . \tag{D1}$$

Define

$$R_{\rm es} = \left[\frac{3}{64\pi^3}\left(\frac{4\pi N_{\rm cl}\dot{M}_{\rm ev}}{\alpha T_h^{5/2}}\right)\left(\frac{\mu}{\mu_{\rm H}k}\right)^{5/2}\frac{E_{\rm th}^2}{\mu_{\rm H}^{1/2}n_0^3}\right]^{1/5} , \tag{D2}$$

which, as shown below, is the radius at which the evaporated mass equals the swept-up mass. Note that $\dot{M}_{\rm ev}/T_h^{5/2}$ is constant; numerically,

$$\frac{4\pi N_{\rm cl}\dot{M}_{\rm ev}}{\alpha T_h^{5/2}} = \frac{\Sigma}{(3.09 \times 10^{18}\ {\rm cm\ pc^{-1}})^3} , \tag{D3}$$

where the evaporation parameter Σ is defined in equation (7), with lengths measured in parsecs, so that

$$R_{\rm es} = 10^{1.19} E_{51}^{2/5} \Sigma^{-1/5} n_0^{-3/5}\ {\rm pc} . \tag{D4}$$

Equation (D1) then becomes

$$\frac{dM}{dx} = 3M_1 x^2 + \tfrac{4}{3} M_1 (M_1/M)^2 x^3 , \tag{D5}$$

where $M_1 = (4\pi/3)R_{\rm es}^3\rho_0$ and $x = R/R_{\rm es}$. An approximate solution can be obtained by iteration when the evaporated mass (represented by the second term) dominates:

$$M = M_1 x^3 + M_1 x^{4/3} ; \tag{D6}$$

or, since $M/M_1 = n_h R^3/n_0 R_{\rm es}^3$,

$$n_h/n_0 = 1 + x^{-5/3} . \tag{D7}$$

The derivation is accurate only for $x \ll 1$, but the result is approximately correct for all x since it has the correct asymptotic limit $n_h \to n_0$ for $x = R/R_{\rm es} \gg 1$. As remarked above, the internal density n_h at $x = 1$ is twice the ambient value n_0, corresponding to equal parts of evaporated and swept-up mass.

Finally, the slowly varying parameter η can be evaluated by differentiating equation (5a):

$$\eta \equiv \frac{d\ln R}{d\ln t} = -\frac{1}{5}\left(\frac{d\ln n_h}{d\ln R}\right)\frac{d\ln R}{d\ln t} + \frac{2}{5} , \tag{D8}$$

where a term proportional to $d(\eta^{2/5})/dt$ has been neglected. The solution of equation (D8) for η is obtained by noting that $d\ln R = d\ln x$ and using equation (D7); the result is equation (6) in the text.

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